Spectral Purity Enhancement via Polyphase Multipath Circuits

Eisse Mensink, Eric Klumperink and Bram Nauta
IC-Design Group, University of Twente
P.O.Box 217, 7500 AE Enschede, The Netherlands
E-mail: e.mensink@utwente.nl

Abstract—**The central question of this paper is: can we enhance the spectral purity of nonlinear circuits by using polyphase multipath circuits? The basic idea behind polyphase multipath circuits is to split the nonlinear circuits into two or more paths and exploit phase differences between these paths to cancel undesired distortion products.**

It turns out that it is very well possible to use polyphase multipath circuits to cancel distortion products produced by a nonlinear circuit. Unfortunately, there are also some spectral components that cannot be canceled with the polyphase multipath circuits.

Keywords—**Distortion, Harmonics, Polyphase, Multipath, Cancellation**

I. Introduction

Transistor circuits can introduce unwanted spectral components due to the nonlinear component characteristics [1]. Linearization by feedback is possible but with limits. Especially in high frequency circuits, the amount of available loop gain is limited. Furthermore, feedback introduces instability risks.

Another commonly used method of removing unwanted spectral components is filtering. However, the achievable Q is limited, impeding steep filtering. Moreover, the required filters are application specific, which is for instance a problem in realizing multi-standard radio transceiver systems.

Thus alternative more flexible techniques for spectral purity enhancement are very much wanted. This paper explores the possibility of polyphase multipath circuits [2] to reject unwanted spectral components. Polyphase multipath circuits are circuits with two or more paths that exploit phase differences between the paths to cancel unwanted signals. Examples with two paths are balanced circuits and circuits for image rejection. This paper takes a generalized look at circuits with two or more paths.

The key aim is to find out to what extend polyphase multipath circuits can cancel distortion. It turns out that most of the unwanted spectral components can be canceled if a sufficient number of paths and phases is used, but there are also some components that cannot be canceled.

The contents of the paper are structured as follows. Section II describes and analyses the polyphase multipath technique. In section III, the results of the analysis of the polyphase multipath technique are compared with some known techniques that cancel unwanted spectral components. Section IV gives the conclusions.

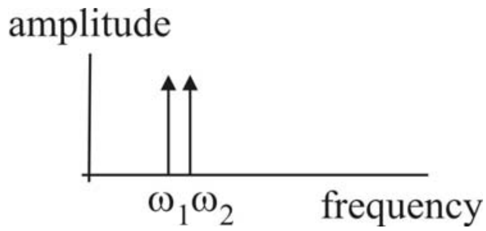

Figure 1a: input spectrum

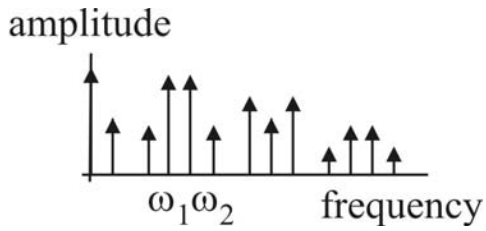

Figure 1b: output spectrum of a nonlinear circuit

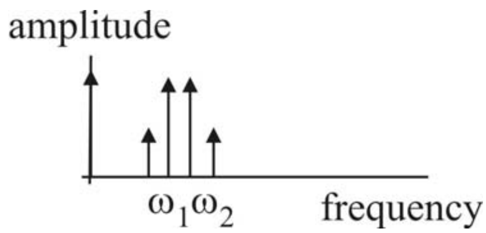

Figure 1c: output spectrum of a polyphase multipath circuit

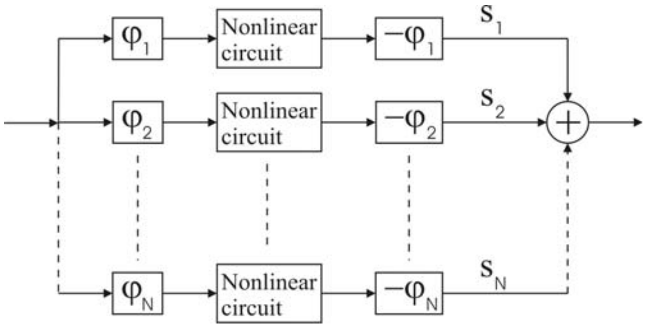

Figure 2: polyphase multipath circuit

II. POLYPHASE MULTIPATH TECHNIQUE

A. Distortion products

Unwanted spectral components often result from nonlinearity, e.g. in RF amplifiers, filters and mixers. A good overview of distortion mechanisms and analysis is given in a paper by Sansen [1]. If x(t) = $A_1\cos(\omega_1 t)$ + $A_2\cos(\omega_2 t)$ is the input of a memoryless nonlinear circuit, the output will not only consist of cosines with a fundamental frequency (ω_1 or ω_2), but also of cosines with other frequencies: harmonics ($n\omega_1$ or $n\omega_2$) and intermodulation products ($k\omega_1+m\omega_2$) (figure 1a and 1b).

The question that we address in this paper is: can we cancel the harmonics and intermodulation products with the help of a polyphase multipath technique (figure 1c)?

B. Polyphase multipath technique

Figure 2 shows a polyphase multipath circuit. The basic idea is to split an input signal into N paths that consist of a phase shifter, a nonlinear circuit with distortion to be canceled, and another phase shifter. The nonlinear circuits are identical, but the phase shifts of the phase shifters differ for every path. At the end, the outputs of the different paths (s_1, s_2, …, s_N) are added to get the output signal.

The aim of the circuit of figure 2 is twofold. On the one hand, the desired signals should have equal phases at the end of every path (figure 3a). In this way they add constructively. On the other hand, the undesired signals (distortion products) should have a different phase at the end of every path and the phase differences between the paths should be chosen in such a way that the unwanted signals are canceled (figure 3b).

How can this situation be achieved? We can find this out by looking at the *phases* of the signals in the different paths. First, we will analyze the harmonics of the input signal, where the first harmonic is the desired signal. Intermodulation products will be looked at afterwards.

C. Phases of harmonics

If we combine the phase shifts of the phase shifters in

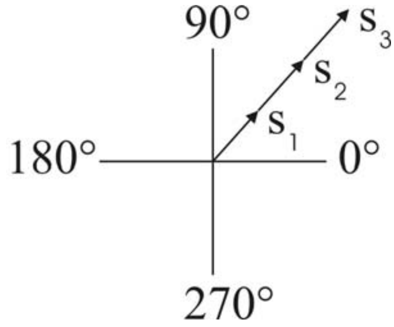

Figure 3a: phases of the desired signals add constructively

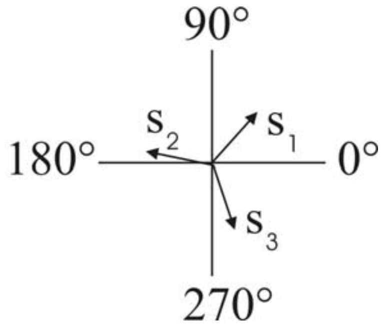

Figure 3b: phases of the undesired signals cancel each other

the different paths into a vector $\underline{\varphi} = [\varphi_1, \varphi_2, \ldots, \varphi_N]$, then the phases of the signals after the first phase shifters are $\underline{\varphi}$ (assuming φ_{in} = 0). The nonlinear circuits generate harmonics of these signals.

We will assume that the circuits are weakly nonlinear and memoryless. Their behavior can be described by a Taylor approximation:

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \ldots \quad (1)$$

where x(t) is the input of the nonlinear circuit, y(t) is the output and the a_i are constants. If x(t) = $A\cos(\omega t+\varphi_i)$, then the desired output will be $a_1 A\cos(\omega t+\varphi_i)$ due to the $a_1 x(t)$ term of equation 1. The other terms in this equation produce the distortion products. $a_2x^2(t)$ will give:

$$a_2 x^2(t) = a_2 (A\cos(\omega t + \varphi_i))^2$$
$$= \frac{1}{2} a_2 A^2 (1 + \cos(2\omega t + 2\varphi_i)) \quad (2)$$

In the same way, $a_3x^3(t)$ gives:

$$a_3 x^3(t) = a_3 (A\cos(\omega t + \varphi_i))^3$$
$$= \frac{1}{4} a_3 A^3 (3\cos(\omega t + \varphi_i) + \cos(3\omega t + 3\varphi_i)) \quad (3)$$

n	0	1	2	3	4	5	6	7	8
Phase of $\underline{s}$	$\underline{0}$	$\underline{0}$	$\underline{\varphi}$	$2\underline{\varphi}$	$3\underline{\varphi}$	$4\underline{\varphi}$	$5\underline{\varphi}$	$6\underline{\varphi}$	$7\underline{\varphi}$

Table 1: phase of the nth harmonic just for the addition in a polyphase multipath circuit

Phase of $\underline{s}$ **N**	$\underline{\mathbf{0}}$	$\pm\underline{\varphi}$	$\pm 2\underline{\varphi}$	$\pm 3\underline{\varphi}$	$\pm 4\underline{\varphi}$	$\pm 5\underline{\varphi}$	$\pm 6\underline{\varphi}$	$\pm 7\underline{\varphi}$
2		C		C		C		C
3		C	C		C	C		C
4		C	C	C		C	C	C
5		C	C	C	C		C	C

Table 2: this table indicates whether harmonics are canceled ("C") or not (blank) depending on their phase (columns) and number of paths (rows), $\underline{\varphi}$ = [0*360º/N, 1*360º/N, …, (N–1)*360º/N]

From these two equations, it is already seen that the phase of the input signal undergoes exactly the same operations as the frequency: if a term with $n\omega t$ is produced, then it will have a phase of $n\varphi_i$.

In general, the phases of the nth harmonic after the nonlinear circuits (in vector notation) will be

$$\underline{\varphi}_{nonlinear,n} = n \cdot \underline{\varphi} \tag{4}$$

The phase shifters after the nonlinear circuits set the phase of the desired signal (n = 1) back to zero. The higher harmonics undergo the same phase shifts (the phase shifters are assumed to be frequency independent). Therefore, the phases of the nth harmonic just before the adder will be

$$\underline{\varphi}_{s,n} = (n-1) \cdot \underline{\varphi} \tag{5}$$

The desired signals will have the *same phase* in every path ($\underline{\varphi}_{s,n} = \underline{0}$) and *add constructively*. The higher harmonics will have a *different phase* in every path ($\underline{\varphi}_{s,n} \neq \underline{0}$) and *can be canceled*. It depends on the choice of $\underline{\varphi}$ and the number of paths, N, whether these harmonics are canceled or not.

A. Canceling of Harmonics

If the phases between the paths are chosen *equidistant*, thus $\underline{\varphi}$ = [0*360º/N, 1*360º/N, …, (N–1)*360º/N], then equation (5) can be rewritten to

$$\underline{\varphi}_{s,n} = \frac{(n-1)}{N} \cdot [0 \cdot 360^\circ,\ 1 \cdot 360^\circ,\ \ldots\ ,\ (N-1) \cdot 360^\circ] \tag{6}$$

This equation shows that whenever n = p*N+1 (p = 0,1,2,…), the phase in every path will be a multiple of 360º leading to constructive addition (figure 3a). It can be shown that in all other cases, so n ≠ p*N+1, the phases are distributed equidistantly over 360º, leading to cancellation (figure 3b). Therefore, for N paths all harmonics are canceled, except the (p*N+1)th harmonics.

The same conclusion can be drawn from tables 1 and 2. Table 1 shows the phase of a certain harmonic (table 1 is produced with the help of equation 5). Then table 2 can be used to see if a harmonic with this phase is canceled or not by a certain number of paths. The table, which is only valid for equidistant phases, shows that only if this phase ($\underline{\varphi}_{s,n}$) is equal to p*N*$\underline{\varphi}$ the harmonic is *not* canceled. Combining this with equation 5 leads to the result that only the (p*N+1)th harmonics are *not* canceled.

In general, the more paths (and phases) are used, the more harmonics are canceled.

B. Canceling of intermodulation products

The canceling of intermodulation products can be analyzed in the same way as the canceling of the harmonics. Similar to equations 2 to 5, it can be shown that intermodulation products at $k\omega_1+m\omega_2$ (k and m can be positive and negative) will have a phase of (k+m)*$\underline{\varphi}$ after the nonlinear circuits and a phase of (k+m–1)*$\underline{\varphi}$ before the adder. The only difference with equation 5 is that n is replaced by k+m. Therefore, if the nth harmonic is canceled, also the intermodulation products at $k\omega_1+m\omega_2$ with k+m = n will be canceled.

Comparable to the results of section II-C and II-D, the only intermodulation products that are *not* canceled are the intermodulation products with k+m = p*N+1. Unfortunately, this means that the intermodulation products that have k+m = 1 can never be canceled, because otherwise the desired signal (n = 1) would also be canceled. The most important intermodulation products that are in this category are third order intermodulation products at $2\omega_1-\omega_2$ and $2\omega_2-\omega_1$. Unfortunately, these products are often a big problem in RF receivers.

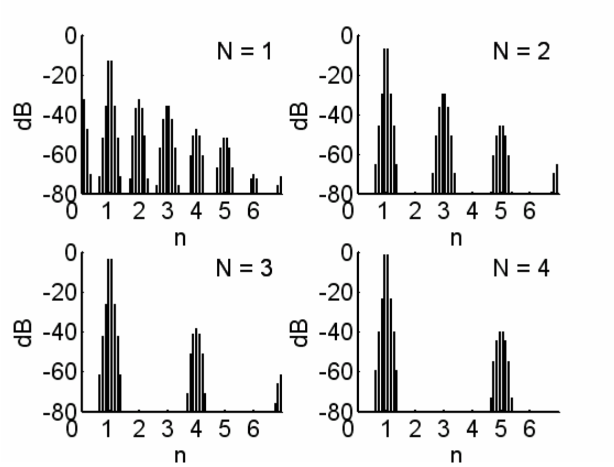

Figure 4: simulated output spectra of polyphase multipath circuits with one, two, three and four paths

Because the intermodulation products with k+m = n will be canceled whenever the n^{th} harmonic is canceled, table 1 and 2 can also be used for intermodulation products: just replace n by k+m. In the remainder of this paper, for briefness we will only discuss the canceling of harmonics.

C. Simulation results

Figure 4 shows the simulation result of a two-tone test. In Matlab, (ideal) phase shifters and nonlinear circuits were implemented and the circuit of figure 2 was built for different numbers of paths. The fourier transform (fft) of the output of the circuits was calculated. On the horizontal axis is the number of the harmonics, n. On the y-axis is the amplitude of the frequency components in dB. The figure shows that the more paths are used, the more harmonics and intermodulation products are canceled. For N paths, the (p*N+1)th harmonics are the only components that are not canceled. Because the frequencies ($\omega 1$ and ω_2) of the two tones are not far apart, the intermodulation products at $k\omega_1+m\omega_2$ with k+m = n appear close to n. Figure 4 shows that when the n^{th} harmonic is canceled, the intermodulation products with k+m = n are also canceled.

D. Mixer as a wideband phase shifter

The phase shifter after the nonlinear circuit has to perform the same phase shift on every harmonic that is in the frequency band of interest. So a phase shifter with a constant phase shift over a wide band of frequencies is needed. Phase shifters are often implemented using RLC filters. However, it is difficult to realize a predefined phase shift over a wide band of frequencies with this type of filters. Also, polyphase filters can be used to generate phase shifted signals [3], but again, it is difficult to maintain a constant phase shift over a wide band of

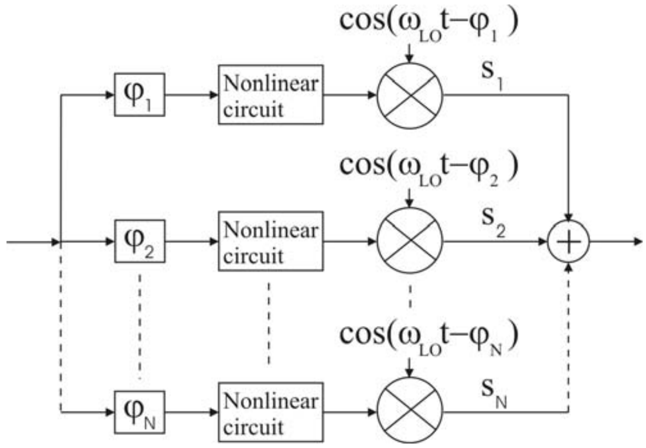

Figure 5: polyphase multipath circuit with mixers as wideband phase shifters

frequencies.

A very wideband phase shifter can be implemented with a mixer [4]. Figure 5 shows the system with mixers. The mixers in the figure are assumed to be ideal mixers (mixer nonlinearity is modeled in the nonlinear circuits). For the frequency band in which the mixer works as a memoryless multiplier, the phase of the local oscillator (LO) signal is added to the phase of the input signal of the mixer, thus introducing a phase shift.

However, the mixer will introduce not only a phase shift, but also a frequency shift. So this technique can only be used if this frequency shift is desired (e.g. in an upconverter).

Furthermore, extra spectral components are introduced: due to sum and difference products of the input signal with the LO signal, both an upper and a lower sideband will appear. So in the frequency spectrum, harmonics will appear at both sides of the LO frequency.

Another problem that can appear in the mixer, is the presence of harmonics of the LO signal (for instance in a switching mixer). These harmonics of the LO signal will also be mixed with the input signal of the mixer and give even more extra spectral components. In the frequency spectrum, the harmonics of the input signal will also appear at both sides of the harmonics of the LO signal.

Assume now, that the desired signal is in the upper sideband, while a positive n means that the harmonic is in the upper sideband and a negative n means that the harmonic is in the lower sideband[1]. Equation 5 will change to

[1] If the desired signal is in the lower sideband, the phases of the LO signals in figure 5 should be made positive instead of negative. Then, a positive n means lower sideband and a negative n means upper sideband.

n / j	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	$5\underline{\varphi}$	$4\underline{\varphi}$	$3\underline{\varphi}$	$2\underline{\varphi}$	$\underline{\varphi}$	$\underline{0}$	$\underline{\varphi}$	$2\underline{\varphi}$	$3\underline{\varphi}$	$4\underline{\varphi}$	$5\underline{\varphi}$
1	$6\underline{\varphi}$	$5\underline{\varphi}$	$4\underline{\varphi}$	$3\underline{\varphi}$	$2\underline{\varphi}$	$-\underline{\varphi}$	$\underline{0}$	$\underline{\varphi}$	$2\underline{\varphi}$	$3\underline{\varphi}$	$4\underline{\varphi}$
2	$7\underline{\varphi}$	$6\underline{\varphi}$	$5\underline{\varphi}$	$4\underline{\varphi}$	$3\underline{\varphi}$	$-2\underline{\varphi}$	$-\underline{\varphi}$	$\underline{0}$	$\underline{\varphi}$	$2\underline{\varphi}$	$3\underline{\varphi}$
3	$8\underline{\varphi}$	$7\underline{\varphi}$	$6\underline{\varphi}$	$5\underline{\varphi}$	$4\underline{\varphi}$	$-3\underline{\varphi}$	$-2\underline{\varphi}$	$-\underline{\varphi}$	$\underline{0}$	$\underline{\varphi}$	$2\underline{\varphi}$
4	$9\underline{\varphi}$	$8\underline{\varphi}$	$7\underline{\varphi}$	$6\underline{\varphi}$	$5\underline{\varphi}$	$-4\underline{\varphi}$	$-3\underline{\varphi}$	$-2\underline{\varphi}$	$-\underline{\varphi}$	$\underline{0}$	$\underline{\varphi}$
5	$10\underline{\varphi}$	$9\underline{\varphi}$	$8\underline{\varphi}$	$7\underline{\varphi}$	$6\underline{\varphi}$	$5\underline{\varphi}$	$-4\underline{\varphi}$	$-3\underline{\varphi}$	$-2\underline{\varphi}$	$-\underline{\varphi}$	$\underline{0}$

Table 3: phase of the n^th harmonic of the input signal mixed with the j^th harmonic of the LO signal just before the addition in a polyphase multipath circuit with mixers as second phase shifters

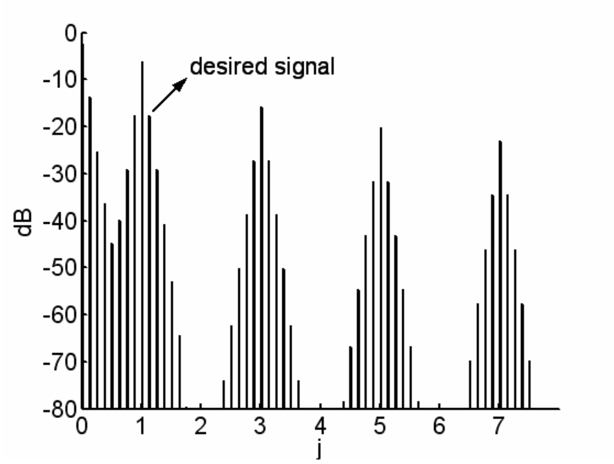

Figure 6: simulated output spectrum of a polyphase multipath circuit with mixers for N = 1 (one path)

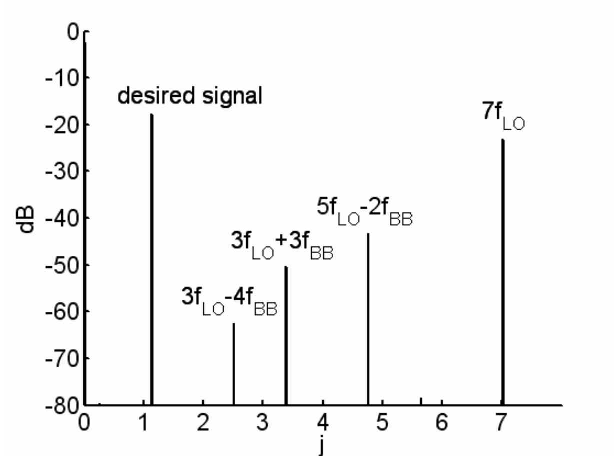

Figure 7: simulated output spectrum of a polyphase multipath circuit with mixers for N = 7 (seven paths)

$$\underline{\varphi}_{s,j,n} = \begin{cases} (n-j)\cdot\underline{\varphi} & n \geq 0 \\ (-n+j)\cdot\underline{\varphi} & n < 0 \end{cases} \tag{7}$$

for the n^th harmonic of the input signal and the j^th harmonic of the LO signal.

Table 3 shows the result of equation 7 for common values of n and j. Table 3 can be used in the same way as table 1: the phase of a certain spectral component at the output can be found in this table. Then, table 2 can be used to see how many paths are needed to cancel that component. It turns out that most of the spectral components can be canceled if a sufficient number of paths is chosen. The only spectral components that the system of figure 5 does *not* cancel are the spectral components with n = j+p*N (p = …,-2,-1,0,1,2,…).

Unfortunately, table 3 shows that not only the desired component (n = j = 1) has $\underline{\varphi}_{s,j,n} = \underline{0}$, but also all other components with n = j (p = 0) have $\underline{\varphi}_{s,j,n} = \underline{0}$, which is independent of N. This means that they cannot be canceled with any number of paths.

A. *Simulation results mixer*

Figure 6 shows a one-tone test in Matlab of a circuit with mixers with one path and figure 7 with seven paths. The spectrums up to the eighth harmonic of the LO signal are plotted. The figures show that exactly those components remain at the output that were predicted by the theory: $3\omega_{LO}-4\omega_{BB}$, $3\omega_{LO}+3\omega_{BB}$, $5\omega_{LO}-2\omega_{BB}$ and $7\omega_{LO}$ (compare with table 3, phases of $p*7*\underline{\varphi}$ will *not* be canceled). Some of these components can be canceled by using a different number of paths. Unfortunately, the components at $3\omega_{LO}+3\omega_{BB}$ cannot be canceled by any number of paths. Other ways have to be found to cancel for instance the third harmonic of the LO signal.

III. Comparison with other techniques

In this section, two known techniques are analyzed using table 1 to 3. It is shown that both techniques can be explained with the help of these tables. However, it is also made clear that the tables are more general. First, balanced (or differential) circuits are examined. After that, the image reject structure as proposed by Gingell [3] is considered.

Balanced circuits are known for their ability to cancel all the even harmonics of a nonlinear circuit. The nonlinear circuit is copied to create two paths. The inputs of the two paths are driven in antiphase and the output is the difference between the outputs of the two paths.

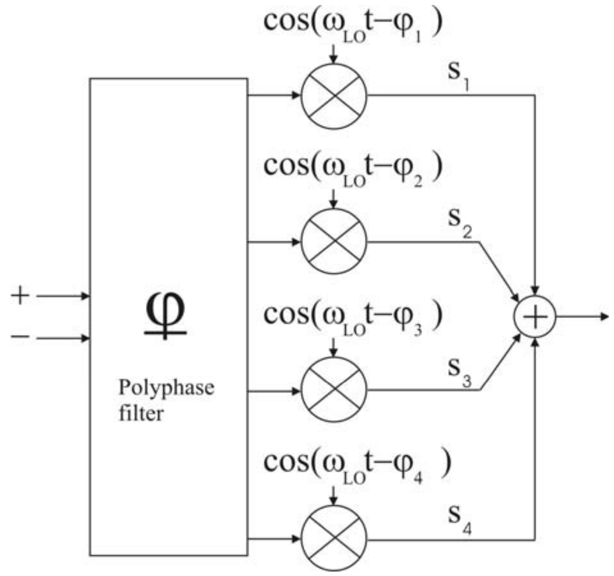

Figure 8: image reject structure of Gingell (N=4)

Comparing this with table 1 and 2, balanced circuits can be considered as polyphase multipath circuits with N = 2, $\underline{\varphi}$ = [0º, 180º]. The phase shifts of figure 2 are performed with minus signs, which are very wideband phase shifters (recall that wideband phase shifters were needed to make the technique work). However, while balanced circuits cancel all the even distortion products, polyphase multipath circuits with more than two paths will cancel even more distortion products.

Of course, just as for balanced circuits, the canceling depends on the matching between the paths. Also, just as for balanced circuits, the signal to noise ratio is not detoriated by the multipath technique.

Gingell [3] proposes to cancel the image signal of a mixer with the help of a polyphase filter before the mixer. He also makes use of multiple paths. Figure 8 shows the system he uses. He uses the polyphase filter to perform the first phase shifts. However, in his work there are no nonlinear circuits, so no harmonics of the input signal are produced.

We can compare his work with table 3 if we look at the columns with n = –1 and n = 1. The image signal (n = –1, j = 1) has a phase of 2$\underline{\varphi}$ and will be canceled by 3 or more paths (it can also be canceled by two paths, but then $\underline{\varphi}$ should be [0º, 90º] instead of [0º, 180º]). Gingell states that not only the image signal, but also the other spectral components up to and including the (N-2)th harmonic of the LO are canceled with N paths. This can also be seen in table 3, looking at the column with n = –1. For example, according to Gingell the first component that is *not* canceled for four paths will be the third harmonic of the LO. Table 3 gives a phase for this harmonic of 4$\underline{\varphi}$. Table 2 shows that this is indeed the first phase that is not canceled with four paths. Table 3, however, does not only show the effect of a polyphase multipath circuit on the harmonics of the LO signal, but also on the harmonics of the input signal, produced by nonlinear circuits.

IV. CONCLUSIONS

It has been shown that with polyphase multipath circuits it is very well possible to cancel distortion products produced by a nonlinear circuit. The more paths and phases are used, the more distortion products are canceled. In this paper tables are presented that can be used to determine whether a certain spectral component will be canceled or not depending on the number of paths. Unfortunately, some intermodulation products (like third order intermodulation products at $2\omega_1-\omega_2$ and $2\omega_2-\omega_1$) cannot be canceled.

If a mixer is used as a phase shifter, extra spectral components appear: components in both an upper and a lower sideband around the harmonics of the local oscillator signal. Most of the components can be canceled by the polyphase multipath circuits if a sufficient number of paths is used. Thus an upconverter mixer can be made with a "clean" output spectrum.

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