Analysis of Multi-Port Transformer in VCO Resonator Design

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Abstract—A multi-port transformer resonator achieves a higher quality factor (Q) than the inductor-capacitor (LC) parallel tank. This paper gives an analysis of the transformer resonator in two different configurations, namely, transformer resonator and series-connected resonator. The results are compared with a simple LC parallel tank in terms of the tank impedance around the resonant frequency. The series-connected resonator consumes the least chip-area, while giving high Q. Three types of VCO using the corresponding resonator circuit are simulated and compared to verify the analysis results.

***Index Terms*—Transformer, Multi-port, Inductance, Quality factor, VCO**

I. INTRODUCTION

THE continuous increases of data rate in modern wireless communication standard is demanded by the large number of users and high bandwidth capacity of the wireless network. This, however, imposes a stringent requirement on the phase noise of voltage-controlled oscillator (VCO). Phase noise of a VCO is inversely proportional to the square of the quality factor (Q) of the resonator tank [1]. By selectively removing the silicon substrate below the inductor [2], shunt losses of the inductive element dramatically reduces and the tank Q increases. High-Q could also be attained by using thick metal layer for the inductor, either by using surface micromachining [3], or by thickening the top-level wiring through local copper electroplating [4].

A transformer could also be utilized in a high-Q resonator design. An effort to double the Q using a differential 2-port transformer resonator was presented in [5]. Following the same principle, a 3-port transformer would triple the Q, and an N-port transformer would increases the Q N-times. An N-port transformer could also be configured as a series-connected of windings to implement an inductor [6]. In a series-connected inductor of N similar layers, the equivalent series inductance increases by the square of N while the equivalent series resistance increases by N. This also gives a Q factor increased of N-times.

This paper presents an analysis of the multi-port transformer in VCO resonator design. Three types of tank circuits: a simple parallel inductor-capacitor (LC) resonator, a 2-port transformer resonator and a 2-port series-connected resonator, respectively, are analyzed and compared in terms of the tank impedance around the resonator frequency. Similar behavior for an N-port transformer could be deduced and proved by additional simulations. To verify the analytical results, three types of VCO using the corresponding resonator circuit are simulated and compared for the same tuning range, oscillation amplitude, supply voltage and power consumption.

II. TWO PORT TRANSFORMER RESONATOR

A. Parallel equivalent inductor model

Integrated inductors and transformers are implemented on-chip using combinations of transmission lines [7], [8]. In this analysis, all the lines parasitic capacitances are assumed to be lossless and lumped to an ideal capacitor in parallel to the inductor. At a given frequency, inductor loss can be represented by a single resistor in parallel with the inductor. With this model, a simple LC tank and a 2-port transformer resonator are shown in Fig. 1.

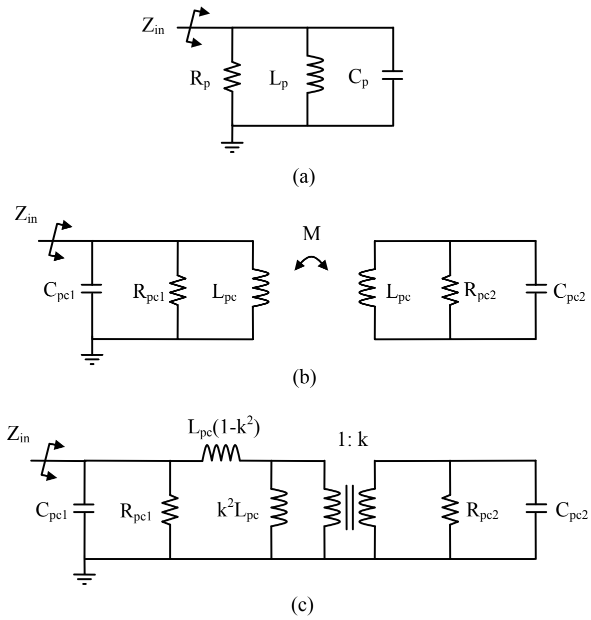

Fig. 1 Resonator tank models: (a) simple LC tank (b) 2-port transformer resonator (c) 2-port transformer resonator with equivalent transformer model

The impedance of the simple LC tank in Fig. 1(a) is

$$Z_{in(1a)} = \frac{sL_pR_p}{s^2L_pR_pC_p + sL_p + R_p} \tag{1}$$

In the transformer resonator given in Fig. 1(b), the two ports are coupled by a mutual inductance M,

$$M = k \times \sqrt{L_{pri} \times L_{sec}} \tag{2}$$

where k is the transformer coupling coefficient, and L_{pri}, L_{sec} are the self-inductances of the primary and secondary windings, respectively. Assuming the same self-inductance value, the tank impedance is derived as (3) and it could be simplified as (4) with an ideal (i.e. k=1) coupling factor. Comparing (1) and (4), the impedances of the LC tank and transformer resonator are identical, with values

$$L_p = L_{pc} \tag{5}$$

$$C_p = C_{pc1} + C_{pc2} \tag{6}$$

$$R_p = R_{pc1} // R_{pc2} = \frac{R_{pc1}R_{pc2}}{R_{pc1} + R_{pc2}} \tag{7}$$

A 2-port transformer resonator with ideal coupling is equivalent to a LC tank circuit. The total resistive loss is the parallel combination of the resistance in each individual port.

The transformer can be represented by an equivalent model as in Fig. 1(c). For k=1, the series leakage inductance $L_{pc}(1\text{-}k^2)$ diminishes and the ideal transformer has a unity conversion ratio. The secondary port is reflected back to the primary, with R_{pc1} in parallel to R_{pc2} and C_{pc1} in parallel to C_{pc2}. This gives the same results as (5)-(7). With non-ideal coupling, the series leakage inductance has to be taken into account.

B. Series equivalent inductor model

While the parallel equivalent model gives the insights of the 2-port transformer resonator, inductor series loss is often modeled by a resistor in series with the inductor [9]. Fig. 2 shows such a series equivalent model of a simple LC tank, a 2-port transformer resonator and a 2-port series-connected resonator. The impedance of the simple LC tank in Fig. 2(a) is

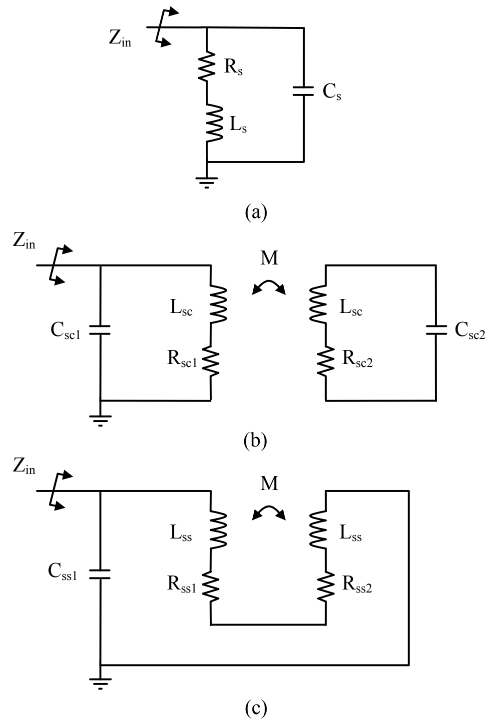

Fig.2 Resonator tanks using series inductor model (a) simple LC tank (b) 2-port transformer resonator (c) 2-port series-connected resonator

$$Z_{in(2a)} = \frac{sL_s + R_s}{s^2L_sC_s + sR_sC_s + 1} \tag{8}$$

The tank impedance of the transformer resonator in Fig. 2(b) is given as (9) and, with an ideal coupling factor, is simplified as (10). The resonant frequency of the transformer tank could be found by the frequency at which the phase shift of the tank impedance is zero [10]. Assuming $C_{sc1}=C_{sc2}=C_{sct}$ and $R_{sc1}=R_{sc2}=R_{sct}$, it could be found that the transformer tank resonates at

$$\omega_{in(2b)}^2 = 1/(2 \cdot L_{sc}C_{sct}) \tag{11}$$

It is similar to the case with the parallel equivalent circuit model. Near the resonant frequency, the equivalent inductance and capacitance are one of the self-inductances, and the sum of

$$Z_{in(1b)} = \frac{sL_{pc}R_{pc1}[s^2L_{pc}(1-k^2)C_{pc2}R_{pc2} + sL_{pc}(1-k^2) + R_{pc2}]}{s^4L_{pc}^2(1-k^2)C_{pc1}C_{pc2}R_{pc1}R_{pc2} + s^3L_{pc}^2(1-k^2)(C_{pc1}R_{pc1} + C_{pc2}R_{pc2}) + s^2L_{pc}[L_{pc}(1-k^2) + (C_{pc1} + C_{pc2})R_{pc1}R_{pc2}] + sL_{pc}(R_{pc1} + R_{pc2}) + R_{pc1}R_{pc2}} \tag{3}$$

$$Z_{in(1b)} = \frac{sL_{pc}R_{pc1} // R_{pc2}}{s^2L_{pc}(C_{pc1} + C_{pc2})R_{pc1} // R_{pc2} + sL_{pc} + R_{pc1} // R_{pc2}} \tag{4}$$

$$Z_{in(2b)} = \frac{s^3L_{sc}^2(1-k^2)C_{sc2} + s^2L_{sc}C_{sc2}(R_{sc1} + R_{sc2}) + s(L_{sc} + R_{sc1}R_{sc2}C_{sc2}) + R_{sc1}}{s^4L_{sc}^2(1-k^2)C_{sc1}C_{sc2} + s^3L_{sc}C_{sc1}C_{sc2}(R_{sc1} + R_{sc2}) + s^2[L_{sc}(C_{sc1} + C_{sc2}) + R_{sc1}R_{sc2}C_{sc1}C_{sc2}] + s(R_{sc1}C_{sc1} + R_{sc2}C_{sc2}) + 1} \tag{9}$$

$$Z_{in(2b)} = \frac{s^2L_{sc}C_{sc2}(R_{sc1} + R_{sc2}) + sL_{sc} + R_{sc1}}{s^3L_{sc}C_{sc1}C_{sc2}(R_{sc1} + R_{sc2}) + s^2[L_{sc}(C_{sc1} + C_{sc2}) + R_{sc1}R_{sc2}C_{sc1}C_{sc2}] + s(R_{sc1}C_{sc1} + R_{sc2}C_{sc2}) + 1} \tag{10}$$

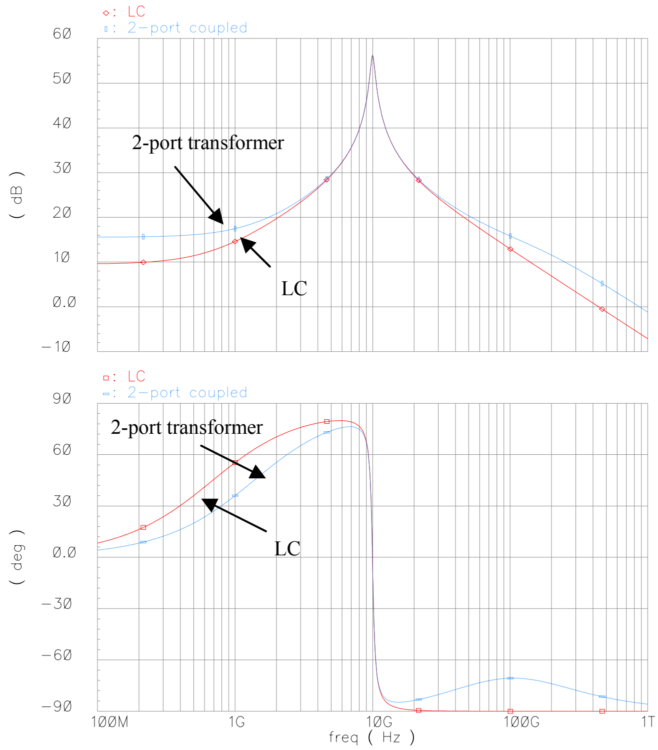

Fig. 3 Impedance of the LC tank and 2-port transformer resonator

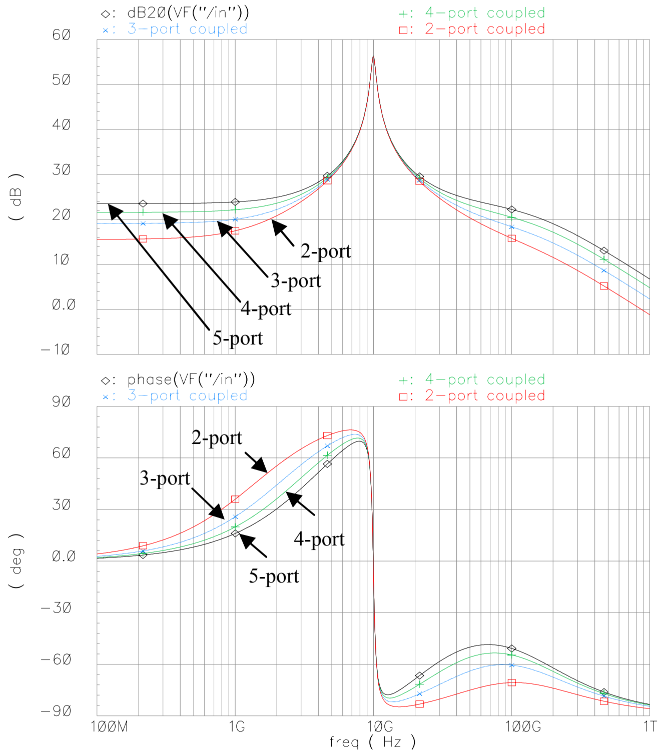

Fig. 4 Impedance of 2-port to 5-port transformer resonators

the primary and secondary port capacitances, respectively. Assuming the total resistive loss is the parallel combination of the resistance in each individual port, the equivalent series resistive loss should be halved.

$$R_{sc} = R_{sct}/2 \tag{12}$$

The tank impedances of Fig. 2(a) and (b) are simulated with resonant frequency set at 10GHz and a tank Q of approximately 7. For the LC tank, L_s=0.7nH, C_s=362fF and R_s=3Ω, while the transformer tank has L_{sc}=0.7nH, C_{sct}=181fF and R_{sct}=6Ω. The result depicted in Fig. 3 shows the two cases match well to each other, both in terms of the impedance at resonance, 3dB bandwidth and slope of the phase response at resonance. Because the transformer resonator has larger resistance and smaller capacitance at the primary port, the out-of-band impedance of the transformer resonator is higher than that of the simple LC tank.

The 2-port series-connected resonator is a parallel combination of a capacitor C_{ss1} and a transformer where the primary port is in series with the secondary port, as in Fig. 2(c). The total inductance L_{sst} contributed by the transformer is simply

$$L_{sst} = 2 \times (L_{ss} + M) \tag{13}$$

and with ideal coupling k=1,

$$L_{sst} = 2 \times (L_{ss} + L_{ss}) = 2^2 \times L_{ss} \tag{14}$$

Thus, the impedance of the series-connected resonator is

$$Z_{in(2a)} = \frac{s(2^2 \times L_{ss}) + (R_{ss1} + R_{ss2})}{s^2(2^2 \times L_{ss})C_{ss1} + s(R_{ss1} + R_{ss2})C_{ss1} + 1} \tag{15}$$

where the inductive series loss is the sum of R_{ss1} and R_{ss2}.

The impedance of the series-connected resonator is simulated with the value L_{ss}=0.175nH, C_{ss1}=362fF and R_{ss1}=R_{ss2}=1.5Ω. Both the simulated magnitude and phase responses exactly overlap that of the LC tank impedance as given in Fig. 3.

To summarized, a 2-port transformer resonator and a 2-port series-connected resonator could have half the equivalent series loss of a simple LC tank, while keeping the same equivalent inductance and capacitance values. Both techniques could double the Q of the resonator.

III. N-Port Transformer Resonator

The tank impedance of an N-port transformer resonator and series-connected resonator could be deduced in a similar manner. For an N-port transformer resonator, with each port consisting of identical self-inductance L_{cN}, series resistive loss R_{cN} and port capacitance C_{cN} (assuming k=1), the equivalent LC model near the resonant frequency is an inductance L_{cN} with series loss R_{cN}/N in parallel with a capacitance NC_{cN}. Impedance of the transformer resonators ranging from two to five ports are simulated with the components values shown in Table I. The resonant frequency and tank Q are maintained at 10GHz and 7, respectively. The results are given in Fig. 4.

For an N-port series-connected resonator with each port consisting of identical self-inductance L_{sN}, series resistive loss R_{sN} (assuming k=1), the equivalent inductance is N^2L_{sN} and the series loss is NR_{sN}. Table II lists the values for two to five port series-connected resonators, keeping a resonant frequency of 10GHz and tank Q of 7. The parallel capacitance C_{sN} is kept constant to be 362fF.

TABLE I Port components of an N-port transformer resonator

	L_{cN}	R_{cN}	C_{cN}
Simple LC	0.7nH	3Ω	362fF
2-port	0.7nH	6Ω	181.0fF
3-port	0.7nH	9Ω	120.7fF
4-port	0.7nH	12Ω	90.5fF
5-port	0.7nH	15Ω	72.4fF

TABLE II Port components of an N-port series-connected resonator

	L_{sN}	R_{sN}
Simple LC	0.7nH	3Ω
2-port	0.175nH	1.5Ω
3-port	77.8pH	1Ω
4-port	43.8pH	0.75Ω
5-port	28pH	0.6Ω

As seen in the table, the inductance to series loss ratio of each individual port inductor decreases by N and thus the corresponding inductive Q of each port decreases by N, while the tank Q of the resonator maintains at the same value of 7. Thus, an N-port transformer increases the tank Q by N-times. For an N-port transformer resonator, the N self-inductances L_{cN} only gives an equivalent inductance L_{cN}. On the other hand, an N-port series-connected resonator having N branches of L_{sN}, gives an equivalent inductance of N^2L_{sN}. The transformer resonator when implemented on a silicon integrated circuit would consume more area for the same self inductance.

IV. N-PORT TRANSFORMER RESONATOR VCO

To verify the analysis results, three types of VCO using: simple LC resonator, two to five port transformer resonators and two to five port series-connected resonators are simulated and compared. All the VCOs have the supply voltage of 1.2V and bias current of 3mA. The VCO negative resistance is driven from a cross-coupled NMOS differential pair with a DC bias current at the common source. The sizes of the NMOS transistors are 5μm/0.12μm using a 0.13μm CMOS process.

In the simulations, the equivalent parallel capacitance and tank Q are the same among all the resonators so that the tuning range and oscillation amplitude are kept to be the same for the comparison. The values of the tank components are listed in Table I and II. The resonant frequency of the resonators is designed to be 10GHz. Including the parasitic capacitance contributed by the active transistors, the oscillation frequency decreases to 9.8GHz. Table III summarized the oscillation frequency, peak-to-peak amplitude and phase noise at 1MHz frequency offset. The matching among all the VCOs proves the validity of the resonator analysis results when applied to a large signal transient VCO simulation.

TABLE III VCO performance of three types of resonator design

	Oscillation frequency	Peak-to-peak amplitude	Phase noise at 1MHz offset
Simple LC	9.77GHz	2.23V	-114.4dBc/Hz
2-port x'former	9.77GHz	2.23V	-114.3dBc/Hz
3-port x'former	9.76GHz	2.24V	-114.2dBc/Hz
4-port x'former	9.76GHz	2.24V	-114.1dBc/Hz
5-port x'former	9.76GHz	2.24V	-114.0dBc/Hz
2-5 port series	9.77GHz	2.23V	-114.4dBc/Hz

V. CONCLUSION

A transformer gives a higher Q factor than a simple inductor in a resonator design. The equivalent tank Q increases by N-times with an N-port transformer. Two different configurations, namely, transformer resonator and series-connected resonator give the same benefit of higher Q, but the transformer resonator would have less equivalent inductance for the same silicon area compared to the series-connected resonator. Theoretical analysis of the two configurations are derived and validated for both small signal and transient simulations.

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