Stochastic Modeling and Reliability Estimation of the Computer Processing Using Markov Chains

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Abstract – **Because the computer processes have a stochastic nature the Markov models are suitable for their mathematical description and analytical investigation. This paper deals with a possibility for analytical description of the system availability using continuous-time Markov chain and evaluation of the reliability level. Two stochastic models for sequential and parallel computer processing are designed and the analytical assessments for reliability level are obtained.**

Key words – **Markov chains; stochastic modeling; computer processing; reliability analysis**

I. INTRODUCTION

As it is known each stochastic process is a family of stochastic values into a common stochastic environment. That process is defined by own distribution function *F(x,t)* and density function *f(x,t)* and it may be discrete or continuous. Analysis and evaluation of different stochastic processes could be carried out by modelling using analytical description [1, 2]. This manner permits to estimate architectural organization and reliability of different computer systems [3, 4]

The computer processes have a stochastic nature and the Markov models are suitable for their mathematical description and analytical investigation [5, 6]. In this connection, the characteristics of system usability, availability and reliability could be estimated by stochastic modeling using discrete-time or continuous-time Markov chains [7, 8, 9].

The purpose of this work is to present possibilities for analytical description of the system availability using continuous-time Markov chain and evaluation of the reliability level. The special features of stochastic modeling at the analytical investigation of computer structures and evaluation of their system availability and reliability are presented. Two stochastic models for sequential and parallel computer processing are designed and the analytical assessments for reliability level are calculated. The obtained results could be used for system reliability optimization and design fault tolerant computer structures.

II. POSSIBILITIES FOR STOCHASTIC MODELING OF COMPUTER STRUCTURES

The stochastic flow is a stochastic process with discrete states and continuous time. The simplest structure of this flow is the regular flow, but the real flows in computer systems are irregular. The Poisson flow in steady-state regime ordinary is used at the stochastic modelling of different computer structures and their workload. The system behaviour could be described as a stochastic process which visits one of the discrete states $\{s_1, s_2, ..., s_n\}$ in the time t. The modelled computer processes usually are akin to the steady-state regime and the description and analytical analysis are based on:

- set of the states $S=\{s_1, s_2, ..., s_n\}$;
- transition probabilities $p_{ij}(t)=P(s_i \to s_j)$ from the state $s_i=S(t)$ to another state $s_j=S(t+1)$;
- initial probabilities $p_i(0)$ for process starting from the state s_i ($i=1, ..., n$);
- final probabilities $p_j(t)/j=1 \div n$ for each state $s_j(t\to\infty)$ which estimate steady-state regime of the process:

$$p_j(t) = \sum_{i=0}^{n} p_i(0).p_{ij}(t) \; .$$

Analytical stochastic modelling is used for investigation and evaluation of computer systems functionality (working capacity, availability, reliability, usability, system performance, etc.). For example, the coefficient of availability could be calculated by $S=\mu/(\lambda+\mu) = (1+\lambda/\mu)^{-1}$ for the steady-state regime, where λ is the faults intensity and μ is the restoration intensity. The function of availability for the transient regime is calculated by:

$$s(i,t) = s + \begin{cases} -\mu(\lambda+\mu)^{-1}.\delta(t); \text{ при } i+0 \\ \lambda(\lambda+\mu)^{-1}.\delta(t); \text{ при } i=1 \end{cases},$$

where $\delta(t) = e^{-(\lambda+\mu)t}$ and two states are defined – fit for work (i=1) and faulted (i=0).

The probability for k faults in a computer structure in the time t could be calculated by $R_k(t)=[(\lambda t)^k/(k!)].e^{-\lambda t}$, where $R_0(t) = R(t) = e^{-\lambda t}$ is the probability for work without faults.

The theory of Markov chains is suitable mathematical tool for an adequate analytical description of the computer processes, because their nature is the same as a Markov's stochastic process. The Markov chain (MC) is a Markov's stochastic process $X(t)$ with a set of states $S=\{s_1, s_2, ...\}$ and the process could visit each state in the time t_{k+1} on the base of a probability that depends only on the previous state in the time t_k (for each sequence $t_1<t_2<...<t_k<t_{k+1}<...$). Usually MC with a finite set of states is used at the stochastic modelling of computer structures and the time could be discrete or continuous value.

The discrete MC (discrete time) defines the probability for visiting of each discrete state $S(k)=s_j$ on the base of the stochastic characteristics of the state in the previous time $S(k-1)=s_i$ by the follow equation for the conditional probabilities $p_{ij}(k)=P[S(k)=s_j/S(k-1)=s_i]$ and k=1,2,... . The behaviour of the process over the set S is described by the sequence $<S(0), S(1),..., S(k),...>$ for separated steps (discrete times) $k=0,1,2,...$ and each possible transition $[S(k\text{-}1)=s_i]\rightarrow[S(k)=s_j]$ could be defined by the formula of the composite probability.

$$p_j(k) \;=\; \sum_{i=1}^{n} p_i(k-1).p_{ij} \;\; (j=1,2,...,n)$$

The continuous MC (continuous time) is more suitable for description of system availability and reliability of computer structures. The modelling organization is the same, but at the continuous time. Each process begins by an initial state $S(0)$ and if in the time $t\geq 0$ it is in the state $S(t)=s_i$, in the following time $t+\Delta t$ the process could go to the other state $S(t+\Delta t)=s_j$ with an intensity λ_{ij}. The transition probability $p_{ij}(t,\Delta t)$ depends on stochastic parameters of the state s_i and eventually on the time. The transition between two states is realized by the first event in the stochastic flow. If the stochastic flow of events is like Poisson's flow the investigated process could be evaluated by continuous MC.

III. RELIABILITY INVESTIGATION OF A SEQUENTIAL PROCESSING STRUCTURE

A conceptual model for stochastic modeling is initiated. It assumes that the computer structure consists of N sequential units U_i (i=1÷N) and we introduce the hypothesis that only one unit may be fault at each moment. It is assumed that all units of the structure could be remounted after their faulting. In this reason, each unit U_i could be in one of the two states – work state (σ_i=0) and state of failure (σ_i=1). The system availability should be described by set of states $s_j=<\sigma_i/i=1\div N>$, for $j=0\div 2^N-1$. The basic state $s_0=<\sigma_1\sigma_2\sigma_3\ldots\sigma_N> = <00\ldots0>$ presents fully worked system and all other states present a system with one or more failures.

Concerning assumed hypothesis the actual set of states for stochastic modeling and investigation system reliability of the structure must be shortened to the N+1 states only – one for the structure in good working order and N for situations of failure in each unit.

On this base a continuous Markov chain with N+1 states $s_j=S(t)\in S$ is designed. This stochastic model could be presented by a "star" type graph of the states that the central node (s_0) is connected with all other nodes by the arcs $\lambda_j(t){:}s_0\rightarrow s_j$ (failure intensity) and $\mu_j(t){:}s_j\rightarrow s_0$ (restoration intensity) where $j=1,2,\ldots,n=N$.

Each fault of unit give rise to transition from the central node (state) s_0 to other node (state) s_j defined by its number j (number of unit in the structure). After each restoration the Markov process returns to the central node (state) s_0. The initial conditions for analytical solving of this Markov model are $p_0(0)=1$ and $p_0(j)=0$ (for $j=1\div N$).

The stochastic analytical model for sequential processing structure description is given below:

$$\left| \begin{array}{l} \dfrac{dp_0(t)}{dt} = \left[\sum_{i=1}^{N} \mu_i p_i(t)\right] - \left[\sum_{j-1}^{N} \lambda_j p_0(t)\right] \\ \dfrac{dp_i(t)}{dt} = \lambda_i p_0(t) - \mu_i p_i(t); \quad i=1,2,...,N \\ \sum_{i=0}^{N} p_i(t) = 1 \end{array} \right.$$

The reliability estimation should be carried out by solving of the following system of equations for an investigation of the steady-state regime of the stochastic processes:

$$\left| \begin{array}{l} \left[\sum_{i=1}^{N} \mu_i p_i\right] - \left[\sum_{j-1}^{N} \lambda_j p_0\right] = 0 \\ \lambda_i p_0 = \mu_i p_i; \quad i=1,2,...,N \\ \sum_{i=0}^{N} p_i = 1 \end{array} \right.$$

The analytical solving of the second equation and after substitution in the third equation should be defined the following assessments for probabilities:

$$p_i = \frac{\lambda_i}{\mu_i} p_0; \quad i = 1,2,...,N$$

$$p_0 + \frac{\lambda_1}{\mu_1} p_0 + \frac{\lambda_2}{\mu_2} p_0 + ... + \frac{\lambda_N}{\mu_N} p_0 = 1 \Rightarrow p_0 = \frac{1}{1 + \sum_{j=1}^{N} \frac{\lambda_j}{\mu_j}}$$

$$p_i = \frac{\lambda_i}{\mu_i \left[1 + \sum_{j=1}^{N} \frac{\lambda_j}{\mu_j} \right]}; \quad i = 1,2,...,N \; .$$

The probability for failure-free work $P(t) = e^{-\int_0^t \lambda(t).dt}$ of the system should be defined by modification of the graph of the states. The new graph will consists the arcs $\lambda_j(t)$:$s_0 \to s_j$ only and it permits to obtain the assessments $P(t) = 1 - \sum_{j=1}^{N} p_j(t)$ and $T=1/\lambda$ (time for failure-free work).

An example of sequential processing structure is presented in fig. 1. It should be modelled by Markov chain with 5 states: s_0=<0000>, s_1=<1000>, s_2=<0100>, s_3=<0010>, s_4=<0001> and vector of the initial probabilities P_0=<1, 0, 0, 0, 0>.

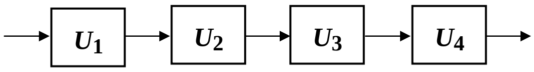

Figure 1

Let us assume that $\lambda_j \neq \mu_j$ and the values for intensities are as follows: $\lambda_1=\lambda_4=\eta$; $\lambda_2=\lambda_3=2\eta$; $\mu_1=\mu_4=2\eta$; $\mu_2=\mu_3=3\eta$. The assessments for stochastic parameters in this case are as follows: p_0=0,3; p_1=p_4=0,15; p_2=p_3=0,2; P(t)=0,3. If it is assumed that all intensities are equivalent and $\lambda=\mu$, the same assessments will be equivalent too – p_0=p_1=p_2=p_3=p_4=0,2.

IV. RELIABILITY INVESTIGATION OF A PARALEL DUAL PROCESSOR STRUCTURE

A dual-processor structure for parallel processing of the tasks is investigated in this part. The tasks has two priorities – the tasks with high priority PR=1 are processed by first processor PU1; the low priority tasks (PR=2) are processed by second processor PU2. If any failure is obtained, the system works as follows:

- if the PU1 is faulted, PU2 will process the high priority tasks and the low priority task will not be processed;
- if the PU2 is faulted, only high priority tasks will by processed by PU1 and any low priority task will be processed by PU1 if this is necessary for a high priority task.

The conceptual model shows a system for parallel data processing with shared memory access (fig. 2). The input stream has a stochastic nature and includes different type tasks that are processed by two priority levels. The main hypothesis for the system work is modeled by a simple stochastic model with tree states – availability (A), task execution (B), faulty (C) and intensities: χ:A$\to$B – of the input stream of the tasks; ν:B$\to$A – of the task processing stream; λ:A$\to$C – of the stream of failures; β:B$\to$C – of the fatal failures during the processing; μ:C$\to$A – of the restoration stream.

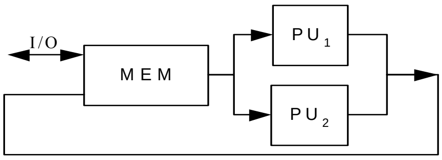

Figure 2

The following analytical model as a system of stochastic equations is designed on the base of the described conceptual model:

$$\begin{cases} \nu.p_A + \mu.p_C - (\chi + \lambda).p_A = 0 \\ \chi.p_A - (\nu + \beta).p_B = 0 \\ \lambda.p_A + \beta.p_B - \mu.p_C = 0 \\ p_A + p_B + p_C = 1 \end{cases}$$

The analytical investigation is connected to the calculation of the final probabilities and it should be carried out in the following sequence.

The probability $p_B = \chi.p_A / (\nu + \beta)$ from the second equation is substituted in the third equation and it is obtained the probability for state C:

$$p_C = \frac{\lambda(\nu + \beta) + \beta\chi}{\mu(\nu + \beta)}.p_A \, .$$

The next steps of the stochastic model solving permit to obtain the following assessments for the final probabilities:

$$p_A = \frac{\mu(\nu + \beta)}{(\nu + \beta).(\mu + \lambda) + \chi(\mu + \beta)}$$

$$p_B = \frac{\chi\mu}{(\nu + \beta).(\mu + \lambda) + \chi(\mu + \beta)}$$

$$p_C = \frac{\lambda(\nu + \beta) + \beta\chi}{(\nu + \beta).(\mu + \lambda) + \chi(\mu + \beta)}$$

Let the p_1 and p_2 are the probabilities for trouble-free processors PU1 и PU2. If the states A and B are working states, but the state C is failure state, the probability for worked structure will be defined as follows:

$$p_1 = p_2 = p_A + p_B = \frac{\mu(\nu + \beta + \chi)}{(\nu + \beta).(\mu + \lambda) + \chi(\mu + \beta)}$$

A numeric example is presented below. Let us assume that the investigated structure can be in one of the states {A, B, C} and vector of the initial probabilities is P_0=<1, 0, 0>. It is introduced the following assumptions:

- The relation between states "availability" and "execution" is 4:3;
- The failures do not depend on the current state of the processing in the structure and their intensity is 1/100 part of the execution intensity;
- The restoration time is 10 times lower than the time for execution of a task.

A numeric solution should be calculated by the following values for the intensities: χ = 75; ν = 100; λ = β = 1; μ = 10. The assessments are shown in the table 1.

Table 1

p_A	p_B	p_C	p_1	p_2
0,522	0,387	0,091	0,909	0,909

V. CONCLUSION

The reliability investigation of computer processing is very important part of the system evaluation. Concerning the stochastic nature of computer processes realized in a discrete structure the Markov chain theory is very suitable means for carry out analytical experiments and obtaining numerical result for evaluation the stochastic parameters. The proposed stochastic models give a possibility for characteristics investigation of the system availability in different cases. The obtained results could be used for system reliability optimization and fault tolerant design of computer structures.

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