Quantum Effects: from Physics to Electronics

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Abstract—**In this paper we discuss the limits of the semiclassical models used till now in solid state physics to describe electronic devices. In particular it is pointed out that quantities such as, for example, mobility, diffusion and characteristic conductivity of materials are obtained through the average of a high number of events. However, in most nanoelectronic devices a single or few quantum events count and therefore these semiclassical models must be considered not applicable anymore.**

After an overview of the main quantum effects for circuits reported in literature, we give a draft of some simple and reliable theories that have been developed to explain these effects and that can be a starting point in order to build basic models for novel nanodevices. Then, we describe some devices that attempt to exploit these effects and aim to overcome the limits that seem to affect the future of CMOS technology.

In relation to these novel devices, also new signal processing and system architecture paradigms are being investigated. For this reason, we will discuss some issues related to the neural paradigm that, as is claimed by many research groups, seems to fully exploit the low-power and high-density integration possibilities offered by nanoelectronic devices and seems to cope with their many sources of uncertainty.

I. Introduction

The semiclassical drift-diffusion models used till now to describe transport of carriers in electronic circuits were mostly obtained through averaging of a high number of microscopic events [1]. This allowed the physicists and electronic engineers to deal with more handy macroscopic quantities. However, due to the shrinking of the dimensions of the devices, a few or even one event determine the behavior of the device (i.e. Single Electron Transistors). Therefore, the quantum mechanical nature of matter is starting to request his attention and these models need coherent extensions.

Let's consider, for example, the conductance of a generic wire. According to Ohm's laws, it can be described macroscopically by the equation:

$$G = \sigma \frac{S}{L} \tag{1}$$

where G is the issued conductance, σ is the characteristic conductivity of the material and S and L are respectively the cross-section and the length of the wire. Considering now nanometric dimensions, some questions rise: what happens if the section of the wire approaches nanometric size? And which is the range of validity of the macroscopic quantity σ, that represents the conductivity of a material? More in general, is this equation able to represent satisfyingly the behavior of a nanoscale conductor?

In a similar way, in semiconductor devices, referring to the drift-diffusion equations used to describe the transport of carriers, we could make the same considerations about quantities such as mobility μ of the carriers and diffusivity D. Indeed these quantities, obtained through accurate measurements, are the result of the averaging of high number of events and thus they will need some reconsideration [2], [3].

In the next paragraph we will describe some experiments that show how the classic view of these quantities is not satisfying for the modeling of the behavior of nanoscale devices. To demonstrate this, we will take as an example the experiments on the Quantized Conductance effect.

II. Quantum Effects: some experiments

The first experiments which reported a quantization of the conductance were performed in the late 80's almost contemporarily by two research groups [4], [5]. Both groups used a similar experimental apparatus that allowed them to build and measure the electrical behavior of a Quantum Point Contact (QPC).

A QPC is a mesoscopic object, on a scale between the macroscopic world of classical mechanics and the microscopic world of atoms and molecules, that has dimensions comparable to the wavelength of the electrons passing through the contact. Because of the fact that the wavelength of the Fermi electrons in a semiconductor is hundred times larger than in metals, it is easier to build a QPC in semiconductors than in metals. It is for this reason that a GaAs-AlGaAs heterojunction was used in these experiments.

The junction has, close to its interface, a two-dimensional electron gas that can be controlled through a top-gated architecture (fig. 1). The top gate is split in two parts that are separated by a small space. Applying a negative voltage to the top gate, it is possible to deplete the electrons below the gate and create a constriction in the electron gas. The width of the constriction can be controlled by the voltage on the gate. The more negative is the voltage, the smaller will be the constriction. If the constriction has dimensions comparable with the wavelength of the Fermi electrons in the electron gas, it realizes a Quantum Point Contact. Thus, applying a small voltage to the contacts placed at opposite sites of the QPC, a current is induced. Measuring this current, the conductance of the QPC can be analyzed. In both the experiments mentioned above, the conductance was found to be quantized in integer units of $G_0 = 2e^2/h$, where e is the charge of the electron and h is the Planck's constant (fig. 2).

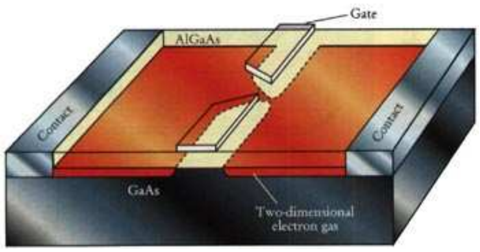

Fig. 1. *Schematic view of a Quantum Point Contact. The external contacts are reservoirs of electrons in local equilibrium that can be used to induce a current.*

After these first experiments, the quantization of the conductance was obtained in plenty of further experiments, also involving metals [6], [7] and Carbon Nanotubes [8] at room temperature . Some of these experiments were even relatively simple [9].

After the discovery of this phenomenon, it was clear that the classic definition of the conductance as the capacity of carrying current in response to an electric field was not satisfying in describing the QPC. A different view was necessary in order to include the quantum nature of matter. In the next paragraph we will describe in general lines what brought the researchers to the view of conductance as transmission.

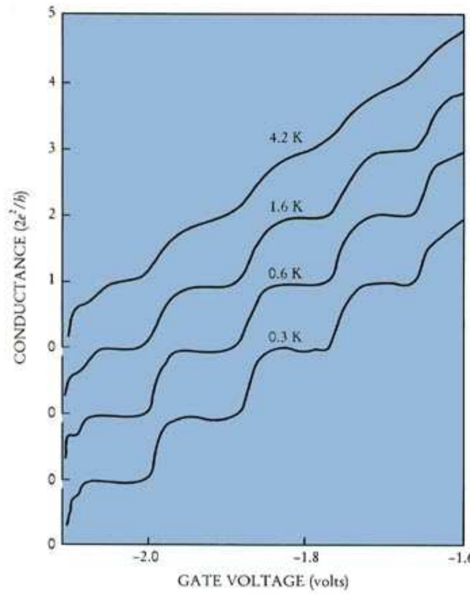

Fig. 2. *Conductance quantization of the QPC. As the gate voltage becomes less negative, the constriction widens continuously but the conductance shows discrete steps. The steps disappear when the thermal energy becomes comparable to the energy separation of the modes.*

III. Theoretical background for new models

The first approach to the view of conductance as transmission was put forward by Landauer in 1957 [10]. However, for a long time, the real implications of this paper where almost ignored. After more than twenty years, the view of conductance as transmission capacity was proposed again by Anderson et al. [11]. This time the debate received more attention and stimulated further discussions [12], [13].

According to Landauer's viewpoint, the conductance was originally expressed as:

$$G = \frac{2e^2}{h}\,\frac{t}{1-t} \tag{2}$$

where t is the transmission probability. However it was underlined by Imry [14] that this formula was giving infinite conductance for t equal to unity because was not taking into account the finite conductance of the contacts. Thus, the formula widely used in the following papers became:

$$G = \frac{2e^2}{h}\,t \tag{3}$$

If the transmission coefficient t is equal to 1 the conductance is equal to G_0. Equation 3 is able to explain the quantization of the conductance reported in the experiments. Furthermore, thanks to this approach, the QPC can be modeled as a electron waveguide. Thus, exactly as in a microwave waveguide, there will be only a finite, integer number of propagating modes associated to discrete energy levels. Using quantum physics calculations it can be shown that the conductance G_0 can be associated to each propagating mode and the total conductance of a QPC with N propagating modes can be expressed as:

$$G = N\frac{2e^2}{h} \tag{4}$$

The rigorous demonstration of this result involves the quantum physic concepts of density of states and group velocity and is not easily comprehensible for a person with an electronic engineering background. Moreover, it is not useful to our purposes.

A simpler explanation that mixes basic quantum physics equations and classical electronic equations has been proposed [9]. This approach can be, in our opinion, a useful compromise in order to describe the mesoscopic features of the QPC from a electronic engineering viewpoint. Here we give a draft of the steps that bring to the desired result.

The current in a one-dimensional wire can be expressed as:

$$I = Ne\frac{v}{L} \tag{5}$$

where N is the number of contributing electrons, v is their average velocity and L is the length of the wire. Then, remembering that the conductance G is the current I divided by the voltage V, we can write:

$$G = \frac{Nev}{LV} \tag{6}$$

The voltage V is the drop of potential energy ΔU of a contributing electron divided by the charge e of the electron. Therefore the conductance becomes:

$$G = \frac{Ne^2v}{L\Delta U} \tag{7}$$

So, the problem of determination of the conductance reduces to the problem of determination of the number of electrons contributing to the current in a one-dimensional wire of length L when the difference of potential energy from end to end is ΔU. To determine this number, we must use some simple quantum mechanics equations associated to the theory of a particle in a box. According to De Broglie equations, in a box of finite length L the wavelengths of the electrons can only assume the discrete values:

$$\lambda_n = \frac{L}{n} \tag{8}$$

and thus the associated velocities will be:

$$v_n = \frac{nh}{Lm} \tag{9}$$

where m is the mass of the electron. In [9] it is now assumed that the number N of contributing electrons can be associated to a range of velocities Δv. Moreover, for the degeneracy of electrons every velocity will have two associated electrons. Thus, using equation 9, we can write:

$$N = \frac{2Lm\Delta v}{h} \tag{10}$$

Remembering that the kinetic energy is $K = \frac{mv^2}{2}$, the increase of kinetic energy for the electrons that contribute to the conduction, neglecting the second order terms, will be:

$$\Delta K = mv\Delta v \tag{11}$$

and using equation 11 in equation 10 we obtain:

$$N = \frac{2L\Delta K}{vh} \tag{12}$$

Finally, substituting equation 12 in equation 7 and remembering that, for the principle of conservation of energy, $\Delta K = \Delta U$, we obtain:

$$G = \frac{2e^2}{h} \tag{13}$$

that is the value that we expected. Then, for every propagating mode, the conductance will be G_0 and, every time the variation of the dimension of a QPC allows the propagation of a new mode, the conductance will increase of a step equal to G_0. Thus, the Quantized Conductance phenomenon can be associated to a waveguide behavior and the electron waveguide can be seen as a transmission line with characteristic admittance G_0.

It is important to notice that the wave-particle duality of electrons is used also in semiconductor devices equations. Therefore, some researchers use this approach to coherently incorporate novel quantum effects in the semiclassical equations [15], [16].

In conclusion, these theories can be a useful starting point in order to build the models that will be necessary to describe and simulate emerging nanoscale devices. In order to give an idea, we will describe shortly some novel devices which make use of Quantized Conductance.

IV. Some circuit applications

In the last decade, the exploitation of Quantized Conductance in nanoscale devices has been investigated by plenty of research groups. The basic idea is to build devices that can be used as electronic switches through the formation and annihilation of nanometric contacts (also called nanowires).

Smith was one of the first to report interesting results [17]. He obtained a switching behavior between tunneling and ballistic transport regimes bringing a sharpened nickel wire into contact with a gold surface. Once again, the conductance in the ballistic regime was found to be G_0. Since today, some other devices have been proposed and investigated [18], [19].

Recently, Terabe et al. [20] built and tested a quantized conductance atomic switch. Through this device, they demonstrated the possibility to control the conductance of atomic bridges between metallic wires (fig. 3). Associating digital states to the presence of these bridges, digital architectures could be implemented. However, although

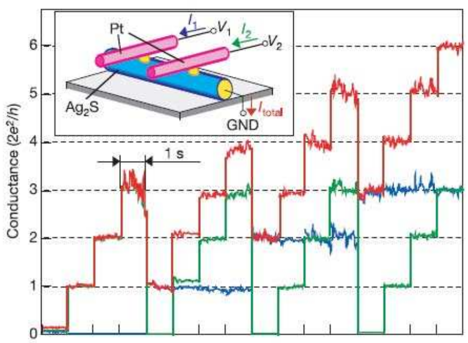

Fig. 3. *Quantized Conductance Atomic Switch. Using voltage pulses it is possible to build nanoscale bridges between the wires and consequently control the conductance of the interconnections*

these applications are very promising, it is also shown that some important problems have to be solved before the Quantized Conductance could be fully controllable. For example, the reproducibility of the results is one of the main issues because of the strong relation of the Quantized Conductance with the particular atomic arrangement.

V. Computing systems and architectures for nanoelectronic devices

At the end of the previous section we pointed out the fact that reproducibility is still a big problem even in more recent nanodevices. This is actually part of a general issue in nanoelectronics. In fact nanoelectronic systems are highly unreliable because of many sources of uncertainty. In the following we will shortly name the main ones.

First of all, the manufacturing process. The technology currently used to build microelectronic devices doesn't allow, at least for now, to extend the well known and widely used top-down manufacturing process to the nanoelectronic field. In particular, the litographic patterning techniques widely used in VLSI CMOS technology have resolutions that are not high enough for nanoelectronic processes. Moreover it seems hardly possible, even in the future, to use this approach in the manufacturing of nanoscale devices [21], [22]. For this reason, novel nanodevices are built through bottom-up self-assembly techniques. However randomness and imprecision are inherent features of these novel techniques and therefore, in large scale integration of nanodevices, the appearance of a relatively large number of defective devices will be unavoidable and permanent errors are introduced during the manufacturing process.

The typically low thresholds involved and the consequent low noise tolerances characteristic of these nanodevices are also sources of uncertainties. In fact, interferences generated, for example, from thermal perturbations or electromagnetic radiations can introduce transient errors during operation mode.

Interconnections are also a big issue. The self-assembly approach makes it difficult to produce precise alignment between wires. Moreover, because of the low drive capabilities of most nanodevices, long distance interconnections must be limited to the minimum.

Actual microelectronic processing systems deal with deterministic quantities and devices with well-defined characteristics. This is not the case in the nanolectronic world. Because of the unavoidable uncertainties of future computers built with nanodevices, new architectures and system paradigms must be investigated. These novel architectures should be fault-tolerant in order to cope with the errors and inaccuracies previously described. For this reason, many researchers claim that the system architectures for nanoelectronic computers will be inspired to neural paradigms [22], [23], [24], [25], [26]. In fact, the human neural system shows high reliability despite of the unreliable and defective units it is made of and, furthermore, it is unsupervised and self-organizing. This reliability seems obtained through a high level of redundancy of similar basic cells and through high adaptation properties that allow compensation or inhibition of faulty cells.

Nanoelectronics will allow to integrate a high number of devices (10^{10}-10^{12} devices on a single die) and therefore will allow, in principle, high levels of redundancy. Moreover, cellular neural networks and neuromorphic systems implement learning and adaptation algorithms in order to adapt the behavior of the network to the task at issue and to minimize errors generated by faulty cells or cells groups. These errors can be represented through distribution of probabilities and expectation values. Thus the design of these networks will reasonably request the intense use of probabilities theory and stochastic modeling. For example, in the desing of some neural networks for nanoelectronics, the use of Markow chains has been helpful [26].

Finally, to front the above mentioned interconnection problems, cellular neural networks with locally connected cells have been proposed. That means that every cell of the network is connected with his neighbors within a fixed radius. These considerations explain also because, in most of the proposed neural networks for nanoelectronics, unsupervised and local learning algorithms are preferred to the supervised ones that request global and long-distance interconnections.

VI. Conclusion

In conclusion, in this article we have shown that the models used until now to describe the behavior of electronic devices cannot be considered applicable as we enter in the nanoelectronic world. To do this we have used as an example the experiments on the Quantized Conductance effect. Then we have reported some simplifications of quantum mechanics equations that have been proposed and that could be useful to build circuit models for devices which exploit this effect. These simplifications are obtained associating the conductive channel with a electron waveguide. This approach is a starting point also for possible extensions, in physics of semiconductor devices, of the semiclassical drift-diffusion equations.

Finally, we explained why neural paradigms seem to be the natural approach in the architectural design of future nanocomputers. Fault-tolerance and adaptation are fascinating properties for nanoelectronic circuits but the intensive use of stochastic modelings will be necessary.

Further work will investigate the herewith discussed solutions and will study new ones in order to reduce the big gap still present between the manufacturing and modeling of nanodevices and the design of efficient and competitive nanocomputers.

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