

# Confidence measures for consensus sets in transformation uncertainty

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*Abstract*—Many applications in image processing require the fitting of transformation models to sets of points. We propose a technique that optimizes the model fitting process by employing parametric consistency as an additional constraint. The technique is used to find affine transformations between data points. The uncertainty about the exact location of a data point is modelled by defining a convex uncertainty region in which the data point must be situated. The uncertainty of a transformation is represented by a convex polytope in the parameter space. For each polytope we introduce a consensus set and a confidence level. The consistency of different parameter polytopes is evaluated with an intersection graph. Consensus sets for fitting models correspond to large cliques in the consistency graph. Not only the size of the consensus set, but also the confidence that we have in it plays an important role. From the size of the uncertainty regions induced by an uncertainty transformation, we compute an expected size of the consensus that would occur by coincidence. These results are used to compute a confidence measure for the consensus set, which results in a further enhancement of the clique finding algorithm.

*Keywords*— transformation uncertainty, transformation polytopes, consensus, confidence measures

## I. INTRODUCTION

Many applications in image processing require the fitting of transformation models to sets of points, by methods such as RANSAC or the Hough transform. The technique that we propose optimizes the model fitting process by employing parametric consistency as an additional constraint. The technique is applied to an important problem: finding affine transformations and homographies between corresponding data points in different images. Therefore we need to determine which are the reliable sets of correspondences.

In what we propose possible correspondences are to be found in confined regions, which reduces the search space for matches considerably. That is, the proposed method employs constraints on the transformation parameters to match corresponding feature point pairs. Furthermore,

we compute a confidence measure for the correspondence sets. This measure is used as a heuristic during the search process. By reducing the transformation parameter space and introducing an heuristic, we can perform better than a full blind search for a random consensus. The most important parameters that we take into account are:

- the size  $\lambda$  of the feature confinement regions;
- the positional inaccuracy  $\epsilon$  of a feature detector;
- the density  $d$  of the feature points in the images, which may include many false positives and false negatives;
- the minimal size  $|C_r|$  of a consensus set  $C_r$  so that the consensus becomes sufficiently reliable.

In this paper we show that without confinement regions we are deemed to take  $|C_r|$  as large as possible, e.g. equal to the number of estimated inliers. With confinement regions we obtain a much better result. Another advantage of the proposed method is that we do not have to introduce an assumed proportion of inliers beforehand. The number of inliers is estimated by the search process itself.

Alternatively, the search space can be reduced by using descriptors for detected affine invariant features [1], e.g. SIFT-descriptors or affine invariant moments. The idea of our method is quite similar, but based upon spatial information instead of local intensity information. A combination of both methods may further improve the results, but will not be discussed in this paper.

The technique has been applied to real outdoor scenes with moving objects in a changing background, so the proposed method must not only find correct correspondence sets but also distinguish between sets corresponding to different objects or the background. Experimental results show that consistency constraints and confidence measures can improve the process of finding correct matches considerably.

Section II describes how transformation uncertainty is modelled in this paper. Section III introduces notions about the reliability of a set of correspondences and Section IV gives experimental results. Finally the paper is concluded in Section V.

## II. UNCERTAINTY REGIONS AND PARAMETER UNCERTAINTY POLYTOPES

We consider the following general matching problem. Let  $S$  be a finite set of source points  $p_i \in \mathbb{R}^2$ . For each point  $p_i$ , let  $Q_i = \{q_{i1}, q_{i2}, \dots\}$  be a finite set of candidate image points  $q_{ij} \in \mathbb{R}^2$ . We look for the optimal affine transformation  $T$  that maps as many source points upon image points as possible.

Hartley and Zisserman give an overview of the cost functions that are used to compute the optimal transformation  $T$  [2]. The cost may either be measured in the image plane (geometric distance), in the source plane (reprojection error) or in both planes. For example, once a possible correspondence map  $q_i = f(p_i)$  has been given between source and image points, with  $q_i \in S_i$ , we can look for the transformation  $T$  that minimizes  $\sum_i (d(p_i, \hat{p}_i)^2 + d(q_i, \hat{q}_i)^2)$ , subject to the condition  $\hat{q}_i = T(\hat{p}_i)$ .

However, fitting parameters to a data set is only part of the problem. A typical matching algorithm first separates the inliers from the outliers among all  $q_{ij}$ , and it searches for a good correspondence map  $f$ . This can be accomplished by a robust estimation method, such as RANSAC. Once the inliers and the correspondence map are known, the algorithm determines an optimal estimation for the transformation  $T$ , by minimizing a cost function [2]. In this paper, we focus on the first step of the algorithm which separates the inliers from the outliers, determines the correspondences, and gives a first approximation for the transformation  $T$ . The goal of this paper is to show that this step can be enhanced considerably if we know additional constraints for the transformation parameters. These constraints can exist either in the form of uncertainty confinement regions for image points, or as  $n$ -dimensional polytopes for transformation parameters (e.g., by giving lower and upper bounds for each of the parameters). Additionally, we want to show how confidence measures on the consensus set can assist in the search process.

Given two images  $I_1, I_2$  with two sets  $S_1, S_2$  of feature points  $p_i \in S_1$  and  $q_j \in S_2$ , the density of the feature points is  $d_i$ , with  $d_i = N_i/wh$ , where  $w$  and  $h$  are the width and height of the image, and  $N_i = |S_i|$  is the number of feature points in the image  $I_i$ .

Suppose we are given a finite set of correspondences  $S_c = \{(p_i, q_j), (p_k, q_l), \dots\}$ ,  $S_c \subset S_1 \times S_2$ , so that there is an affine transformation mapping the points  $p_i$  close to the points  $q_j$ . Then how do we know that this set of correspondences is reliable? The given set of correspondences may also be incomplete. What are the chances of  $S_c$  being part of a larger set of reliable correspondences? Actually, we may start from a single pair  $S_c = \{(p_i, q_j)\}$  and ask

whether it is likely that this pair is part of a large set of reliable correspondences. If we would have some measure for how promising the pair  $(p_i, q_j)$  is, this would certainly facilitate the search process.

In this paper we introduce the inaccuracy  $\epsilon$  of the feature detector, the size  $\tau$  of a confinement region and we will show that these two parameters and the density  $d$  of the feature points are very important to estimate the reliability of the proposed set of correspondences. We shall derive an expression for the probability that a given set of correspondences can arise by accident. But first we have to introduce models for the uncertainty of a position and a geometric transformation.

**Uncertainty of a position.** To model the uncertainty of the image  $(x', y')$  of a point  $p = (x, y)$  in  $\mathbb{R}^2$ , we define the uncertainty region  $R$  as a convex polygon bounded by  $n$  halfplanes,

$$r_i x' + s_i y' \geq 1, \quad 1 \leq i \leq n \quad (1)$$

in the  $x'y'$ -plane. The uncertainty regions  $R$  used in this work will be rectangles, which is sufficient for most applications.

**Uncertainty of a transformation.** We let  $\tilde{T}(p, R)$  denote the set of all transformations  $T$  that map  $p$  into the uncertainty region  $R$ . In this work, to simplify some of the definitions, we restrict ourselves to affine transformations of the form

$$\begin{aligned} x' &= ax + e \\ y' &= dy + f \end{aligned} \quad (2)$$

with  $(x', y')$  as image-point,  $(x, y)$  as source-point and the 4 parameters of the transformation:  $a, e, d, f$ . Note that the restriction to 4 parameters instead of the usual 6 of a general affine transformation is not essential from the theoretical viewpoint, but enables us to simplify notation, and reduces the search space. Most of the results in this paper can be extended to more general transformations in a straightforward manner.

The transformations  $\tilde{T}(p, R) : p \rightarrow R$  form a convex polyhedron  $\tilde{T}$  in 4 dimensions, supported by the hyperplanes that can be found by substituting the equations (2) in (1), yielding

$$r_i(ax + e) + s_i(dy + f) \geq 1, \quad 1 \leq i \leq n. \quad (3)$$

The notion of a transformation polyhedron can now be defined for a given set of correspondences  $S_c = \{(p_i, q_{ij}), (p_k, q_{kl}), \dots\}$ ,  $S_c \subset S_1 \times S_2$ . First the global constraints on the transformation are taken into account. The transformed image of each point  $p_i$  is confined to a

region  $R_i$ , where  $R_i$  are squares of height  $\tau$  centered on each point  $p_i$ . Thus the transformation must be contained in  $\tilde{T}_g = \cap_i \tilde{T}(p_i, R_i)$ ,  $p_i \in S_1$ . Next, we take into account the constraints that follow from the correspondences. For each correspondence  $(p_i, q_{ij})$ , we define a small square  $R_{ij}$  of height  $\epsilon$ , with  $\epsilon < \tau$  centered on  $q_{ij}$ . We define the polytope  $\tilde{T}_{ij} = \tilde{T}_g \cap \tilde{T}(p_i, R_{ij})$ . Finally, for the entire set of correspondences we define the polytope

$$\tilde{T}(S_c) = \cap_i \tilde{T}_{ij}, (p_i, q_{ij}) \in S_c.$$

The polytope  $\tilde{T}(S_c)$  comprises all affine transformations that map each point of  $S_1$  into a square of size  $\tau$  around it, and that in addition map each point  $p_i$  that occurs in  $S_c$  into a smaller square centered on  $q_{ij}$ .

When  $S_c$  contains at least 2 pairs of points the polyhedron  $\tilde{T}$  is bounded, and becomes a convex polytope of transformations in 4-dimensional space.

**Positional uncertainty implied by transformational uncertainty.** For a given set of correspondences  $S_c$  and the transformation polytope  $\tilde{T}(S_c)$  that can be derived from it, it is important to know whether other pairs of feature points can be added to  $S_c$ . Let  $p_m$  be any point in  $S_1$  and  $\tilde{T}$  a given polytope.  $R(p_m, \tilde{T})$  denotes the implied uncertainty region resulting from mapping  $p_m$  by the transformations in  $\tilde{T}$ ; that is,  $R(p_m, \tilde{T}) = \{q \in \mathbb{R}^2 : q = T(p_m) \text{ for some } T \in \tilde{T}\}$ . We can show that  $R(p_m, \tilde{T})$  is the convex hull of the points  $T_k(p_m)$  where the transformations  $T_k$  denote the vertices of the polytope  $\tilde{T}$  [3]. From the above definitions it follows that the implied uncertainty regions satisfy  $R(p_i, \tilde{T}_g) \subseteq R_i$ , as well as  $R(p_i, \tilde{T}_{ij}) \subseteq R_i$ .

Now suppose that for some implied region  $R(p_m, \tilde{T})$  we find that there is a feature point  $q_{mn} \in R(p_m, \tilde{T})$ . Then we must have  $\tilde{T}(S_c) \cap \tilde{T}(p_m, R_{mn}) \neq \emptyset$ . In other words, if we extend the set of correspondences with the pair  $(p_m, p_{mn})$ , then for the extended set we still have a transformation polytope  $\tilde{T}(S_c \cup \{(p_m, p_{mn})\})$  which is non-empty.

**Consensus sets.** It is useful to know whether a given set of correspondences can be extended more than once. Therefore, with each transformation polytope  $\tilde{T}$ , we associate a consensus set  $C(\tilde{T})$ . Let  $S_1$  be a finite set of points  $p_i \in \mathbb{R}^2$ , and for each point  $p_i$ , let  $Q_i = \{q_{i1}, q_{i2}, \dots\} \subseteq S_2$  be a finite set of points  $q_{ij}$  associated with  $p_i$  and let  $\tilde{T}$  be a transformation polytope. Then the consensus set  $C(\tilde{T})$  of the polytope  $\tilde{T}$  is defined as  $C(\tilde{T}) = \bigcup_i (R(p_i, \tilde{T}) \cap Q_i)$ . Thus to find the consensus set  $C(\tilde{T})$ , we must compute the implied image regions  $R(p_i, \tilde{T})$ , and find the points  $q_{ij}$  contained in the implied regions. Note that a consensus set may list some points of  $S_1$  more than once, for example, we may have  $C = (p_1, q_{11}), (p_1, q_{12}), \dots, (p_2, q_{21}), \dots$ . Clearly, we have  $S_c \subseteq C(\tilde{T}(S_c))$ , provided  $\tilde{T}(S_c) \neq \emptyset$ .

For each consensus set  $C$  we define a consensus measure  $f(C)$ . It counts the number of points in  $S_1$  that occur at least once in  $C$ . That is,  $f(C)$  is the cardinality of the set  $A = \{p_i, \dots\}$ , such that for each point  $p_i$  in  $A$ , there is at least one pair of the form  $(p_i, \dots)$  in  $C$ .

**The matching problem restated.** We can now restate our interpretation of the matching problem. We want to find a parameter polytope  $\tilde{T}$ ,

- with a high consensus measure  $f(C(\tilde{T}))$ ;
- that sufficiently constrains the transformation;
- with a consensus that did not arise by accident.

The first two requirements can be handled by the implied regions, first by counting the feature points contained in the regions, and second, by looking at the size of the regions. The reliability of a consensus must still be determined, however. To this end, we will compute the probability  $P(|C|)$  that a consensus of size  $|C|$  or larger arises by accident. In this paper we show that the probability  $P(|C|)$  depends strongly on the inaccuracy  $\epsilon$  of the feature detector and the size  $\tau$  of the confinement region. The probability of a misclassification will be small when both  $\tau$  and  $\epsilon$  are small.

In previous work [4] we have shown how we can find good sets of correspondences by finding large cliques in the intersection graph for the polytopes  $\tilde{T}_{ij}$ , that is, how we can find a set of correspondences  $S_c$ , such that  $f(C(\tilde{T}(S_c)))$  is maximal. In fact, while using an intersection graph, we can look for an intersection  $T_{ij} \cap T_{kl} \cap \dots$  over a large number of transformation polytopes that is non-empty. When this intersection is non-empty, we have found a set of correspondences  $S_c = \{(p_i, q_{ij}), (p_k, q_{kl}), \dots\}$ , such that  $\tilde{S}_c = T_{ij} \cap T_{kl} \cap \dots \neq \emptyset$ , and  $f(C(S_c)) = f(C(T_{ij} \cap T_{kl} \cap \dots))$  is equal to or larger than the size of the clique. In this paper we will examine how reliable such a set  $S_c$  is.

### III. RELIABILITY OF A PROPOSED SET OF CORRESPONDENCES

In this section we consider the probability of the occurrence of an accidental consensus set of features in the test image. Suppose we are given a set of correspondences between two sets of feature points. Then what is the probability that such an accidental set can arise when the feature points are distributed randomly over the entire image according to a uniform distribution?

**Estimate for a correspondence set consisting of a single pair.** Suppose we are given a corresponding set consisting of a single pair,  $S_c = \{(p_i, q_{ij})\}$ . When we compute the consensus measure  $f_c = f(C(\tilde{T}(S_c)))$ , we want to know whether a high value for  $f_c$  can also occur by accident, that is when the feature points are randomly distrib-

uted over the image.

The value of  $f_c$  will depend strongly on the size of the implied regions. Figure 1 shows the implied regions for the polytope  $\tilde{T}_{11}$ . The implied regions are rectangles whose size varies over the image. We can show that the average height (or width) of the implied regions of  $\tilde{T}(S_c) = \tilde{T}(p_i, R_{ij}) = \tilde{T}_{ij}$  is bounded from above by

$$\delta = \frac{\epsilon + \tau}{2}. \quad (4)$$

We briefly sketch the proof of this bound on the size of an implied region. First, we note that the width or the height must increase piecewise linearly, according to either the value of the  $x$  or  $y$  coordinate. It is sufficient to note that the implied region of a point  $p$  is the convex hull of the points  $T_k(p)$ , where the transformations  $T_k$  are the vertices of the transformation polytope [3]. To be specific, suppose we have a transformation polytope with vertices  $(a_i, e_i, d_i, f_i)$ . Then the implied regions have vertices  $(a_i x + e_i, d_i y + f_i)$ . Hence the width of the region varies along the  $x$ -direction as  $(a_i x + e_i) - (a_j x + e_j)$ . In fact, one can show that the width of the implied regions varies as a function that has a bow tie shape, similar to what we have for the collinearity of digital straight lines or the uncertainty cones for point-line coincidences [5, 6]. This linear variation can also be seen in Figure 1. Finally, we know that the smallest width for the implied region is  $\epsilon$  and the largest size is  $\tau$ . It follows that the average width of the implied region is bounded from above by  $\delta = (\epsilon + \tau)/2$ .

Let  $R(p_k, \tilde{T})$  be the implied region of an arbitrary feature point  $p_k$  in the first image. We assume that all the other feature points in each image are distributed randomly over the entire image. Thus the probability that a feature point in the second image lies in this implied region is equal to the ratio  $\delta^2/(wh)$  with  $w$  and  $h$  respectively the width and the height of the image. The second image contains  $N_2$  feature points. Hence the probability  $P_s$  that for a given point  $p_k$  there is at least one feature point  $p_{kl}$  in the implied region  $R(p_k, \tilde{T})$  is equal to

$$P_s = 1 - \left(1 - \frac{\delta^2}{wh}\right)^{N_2}.$$

If the implied region contains at least one feature point, the consensus measure  $f_c$  increases by one. The first image contains  $N_1$  feature points. What is the probability  $P(g)$  that we have  $f_c = g$ ? First, we note that the consensus set contains already the pair  $(p_i, q_{ij})$ . Hence the probability is given by the binomial distribution that for the remaining  $N_1 - 1$  points in  $S_1$ , there are exactly  $g - 1$  points that add

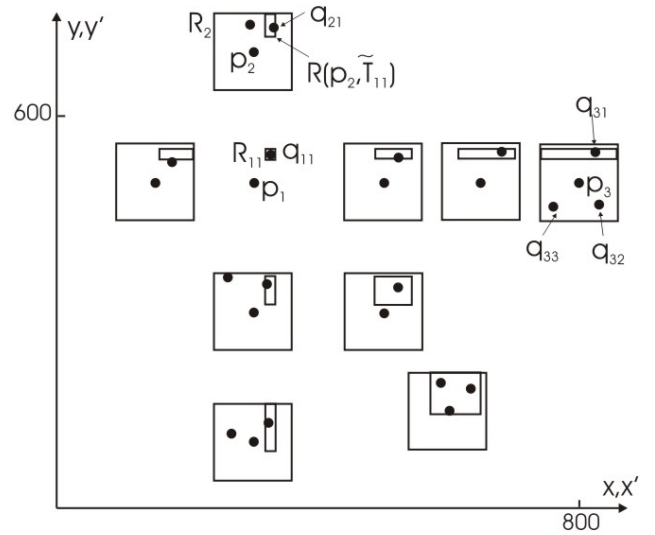


Fig. 1. To determine the polytope  $\tilde{T}_{11}$ , we introduce large squares  $R_2, \dots$  for the points  $p_2, \dots$ , and a small square  $R_{11}$  centered around a candidate point  $q_{11}$ . The consistency polytope  $\tilde{T}_{11}$  then consists of all transformations that map each point  $p_i$  into the corresponding region  $R_i$ , and  $p_1$  into  $R_{11} \subseteq R_1$ . With  $\tilde{T}_{11}$  we compute the implied regions  $R(p_i, \tilde{T}_{11})$ , e.g. the region  $R(p_2, \tilde{T}_{11})$ .

one to the consensus measure  $f_c$ ,

$$P(g) = \binom{N_1 - 1}{g - 1} P_s^{g-1} (1 - P_s)^{(N_1 - g)}, \quad (5)$$

for  $1 \leq g \leq N_1$ . Thus the probability  $P(g)$  depends solely on the sizes of the rectangles  $(\epsilon, \tau)$ , on the number of feature points  $(N_1, N_2)$ , on the width  $(w)$  and height  $(h)$  of the images, and on the value of the consensus measure  $(g)$ .

The probability that we have  $f_c \geq g$  is equal to

$$P(f_c \geq g) = 1 - \sum_{0 \leq i \leq g-1} P(i).$$

The expected mean value for the consensus measure is

$$m = N_1 P_s.$$

Figure 2(a) shows the binomial distribution  $P(g)$  as given by Eq. (5) for  $N_1 = 19$ ,  $N_2 = 162$ ,  $\epsilon = 6$  and for three different values for  $\tau$ ,  $\tau = 40, 80, 120$ . These values correspond to the number of features and the parameters that have been extracted in Figure 5. Figure 2(a) shows how the distribution depends on the value of  $\tau$ . If we can confine the feature points to a relatively small region, e.g.  $\tau = 40$ , then the probability that  $f_c$  is larger than 10 just by coincidence becomes very small. On the other hand, the probability also depends on the value of  $\epsilon$ . When  $\epsilon$  gets larger (a less precise feature detector), also

$P(g)$  gets larger, and the consensus of a correspondence set becomes less reliable.

Figure 2(b) shows the real frequency histogram of the consensus measure  $f_c$ , where the size of the confinement regions was set to  $\tau = 80$ . The values that occur are larger than what was predicted in Figure 2(a) for a random distribution of feature points. They also correspond to a bimodal instead of a binomial distribution. However, the real feature points are not randomly located at all. Since many feature points  $q_{ij}$  in the second image correspond to affine transformations of feature points  $p_i$  in the first image, the consensus measure  $f_c$  of a correspondence set  $S_c = \{(p_i, q_{ij})\}$  will often be larger than what is expected from a random distribution of feature points. Thus the histogram in 2(b) is bimodal. A first peak is due to random feature points that satisfy a binomial distribution, a second peak is due to feature points that correspond to affine transformations. The histogram in 2(b) shows that  $f_c$  can give us some indication about how promising a correspondence set  $S_c = \{(p_i, q_{ij})\}$  is, even when it still consists of a single pair.

**Estimate for a correspondence set consisting of a single pair when the implied regions are known.** The probability in Eq. (5) is an estimate when the exact sizes of the implied regions are unknown. For a real image, we can compute the implied regions, and thus we can give a better estimate for the average value of their size  $\delta$ . For the features extracted in Figure 5, the average value of all the implied regions  $R(p_i, R_{ij})$  taken over all possible pairs  $(p_i, q_{ij})$  was  $\delta = 30$ , instead of the estimated value  $\delta = 43$ , which came from Eq. (5). Figure 2(c) shows the binomial distribution  $P(g)$  as given by Eq. (5) for  $N_1 = 19$ ,  $N_2 = 162$ ,  $\epsilon = 6$ ,  $\tau = 80$ , and where we have taken  $\delta = 30$ , instead of the estimated value given by Eq. (4), which was used for the general case.

**Estimate for a correspondence set consisting of two pairs.** The calculation can easily be redone for a correspondence set containing two pairs,  $S_c = \{(p_i, q_{ij}), (p_k, q_{kl})\}$ . The major difference is that the size of the implied regions will be much smaller. The transformation polytope is  $\tilde{T}(S_c) = \tilde{T}_{ij} \cap \tilde{T}_{kl}$ . In general this polytope will be considerably smaller than the polytopes that come from a single pair, and therefore the implied regions  $R(p_m, \tilde{T}(S_c))$  will also be smaller. A precise analytical estimate or upper bound for the average size of the implied regions is not known at present. Simulations show that we always find a value smaller than

$$\delta = \frac{\epsilon + \tau}{4}. \quad (6)$$

However, for a real image it is always possible to compute

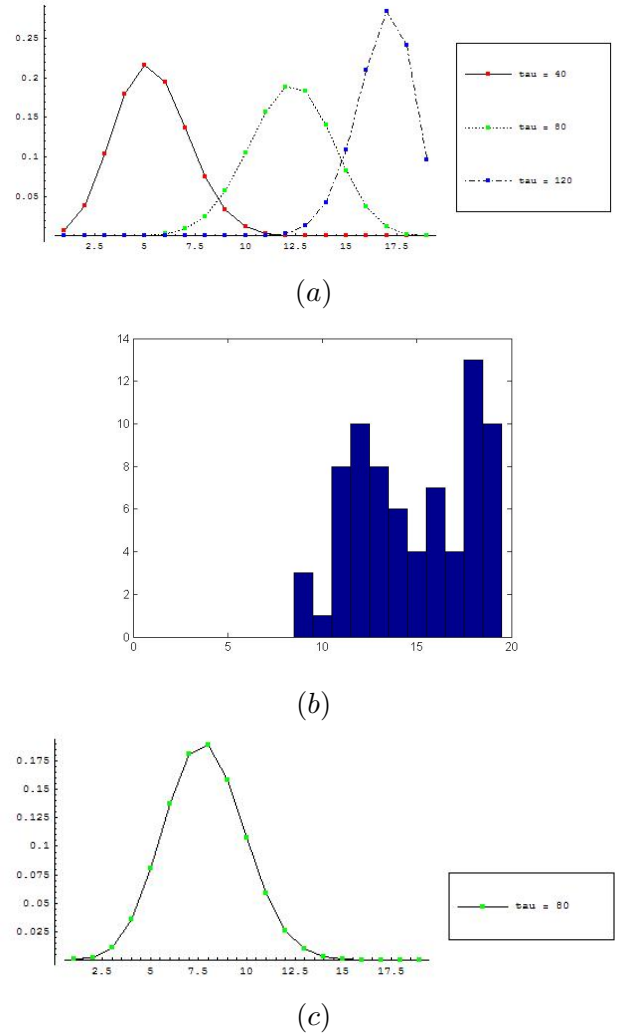


Fig. 2. Estimated distributions (a) and (c) and frequencies for  $f_c$  (b) for a correspondence set containing a single pair.

the exact sizes of the implied regions. Hence, it is perfectly possible to obtain a good estimate for the probability  $P(g)$  that we have  $f(C(\tilde{T}(S_c))) = g$ . As before, we find a binomial distribution,

$$P_2(g) = \binom{N_1 - 2}{g - 2} P_s^{g-2} (1 - P_s)^{(N_1 - g)}, \quad (7)$$

for  $2 \leq g \leq N_1$ , where the probability  $P_s$  is still given by

$$P_s = 1 - \left(1 - \frac{\delta^2}{wh}\right)^{N_2}.$$

Figure 3(a) shows the binomial distribution  $P_2(g)$  as given by Eq. (7) for  $N_1 = 19$ ,  $N_2 = 162$ ,  $\epsilon = 6$ ,  $\tau = 80$ , and where we have estimated  $\delta$  as in Eq. (6). This value for  $\delta$  was obtained during simulations as the average size of the implied regions of  $\tilde{T}_{ij} \cap \tilde{T}_{kl}$ , taken over all possible pairs  $\tilde{T}_{ij}, \tilde{T}_{kl}$  whose intersection was non-empty. Figure 3(b) shows the real frequency histogram of the consensus

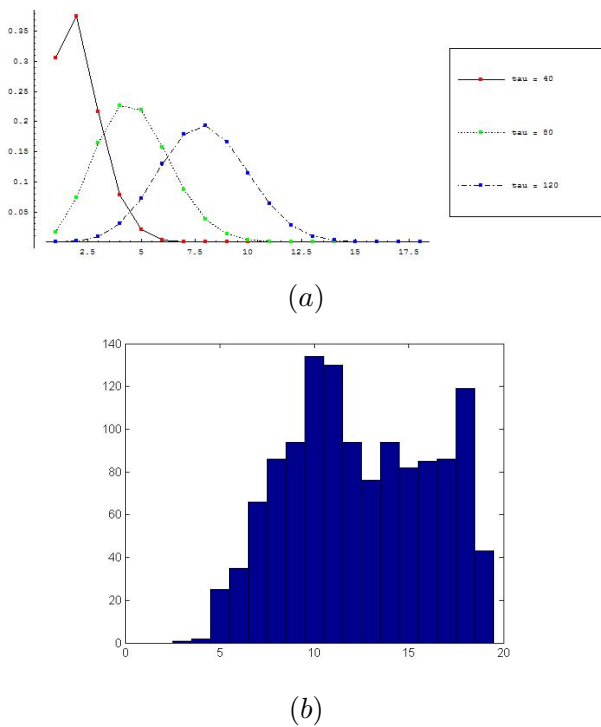


Fig. 3. Estimated distributions (a) and frequencies for  $f_c$  (b) for a correspondence set containing two pairs.

measure  $f_c$  for correspondence sets that contain two pairs. The size of the confinement regions was set to  $\tau = 80$ . Note that the distribution has moved to left (smaller values of  $f_c$ ), and that it has widened. This facilitates the discrimination between promising correspondence sets, and sets whose consensus just arose by accident. Thus the consensus measure  $f_c$  give us a useful indication about how promising a correspondence set  $S_c = \{(p_i, q_{ij}), (p_k, q_{kl})\}$  is. This has been confirmed by the experiments on real images.

#### IV. EXPERIMENTAL RESULTS

We applied our technique to compute the transformations for real outdoor scenes with moving objects: we are interested in the movement of vehicles in image sequences of traffic situations obtained from a moving car. We wish to distinguish between the (moving) background and other vehicles, so we must look for different cliques in the intersection graph. The experimental results show that consistency constraints and confidence measures are of great aid in finding correct matches of different objects.

Figure 4 shows an example of a traffic situation, frame (b) (the test image) is recorded 0.12s later in the sequence than frame (a) (the reference image). The features shown in image (a) of Figure 5 are detected using the Harris corner detector [7] (the original frame is brightened for

clarity). In the other frame, we apply the feature detector within the bounds of uncertainty regions centered on the position of the reference features. The size of the region can be chosen according to the speed of the car from which the pictures are taken and the relative movement of other moving vehicles. Often bounds for these parameters are known or can be estimated.

Typically, we use up to 40 distinct reference features, which results in a feature point density  $d_1$  of  $\frac{40}{480 \times 640}$  in the reference image. The feature density in the second image is higher as there are up to ten false positives for a reference feature; in this case  $d_2 = \frac{N_2}{480 \times 640}$ . Note that there is also a considerable number of false negatives to be found in the reference frame.

The described technique is used to first discriminate the maximum clique in the intersection graph, computed for the situation given in Figure 5. The consensus set for the transformation obtained for this clique is computed and all reference features  $p_i$  and their candidate correspondences are removed for a next calculation. Then again a maximum clique is determined for this new situation with a reduced number of features in both images. Figure 6 shows the two sets of corresponding features, first the features in the reference frame *a*. Frame *b* shows the two sets of correspondence pairs in the consensus set in different symbols. These results clearly show how the features on the moving vehicle in front are distinguished from those on the buildings and the road in the background.

The consensus sets for the results above are larger in size than what could occur by coincidence, as computed by the measures in Section III. The size  $|C_r|$  of the consensus set for which the distributions and the frequencies are shown in Figures 2 and 3 is 16. The probability that a consensus set of this arose by accident is actually rather low:  $P(|C_r| < 0.01)$ . The confidence in this consensus set is proven by the results shown in Figure 6. Note that the correspondence sets shown in this figure are of smaller size due to refinement of the transformation between the corresponding points.

#### V. CONCLUSION

We proposed a simple method to distinguish reliable correspondences among two sets of features by demanding transformation parameter consistency. Employing consistency constraints allows us to determine a consensus set for each uncertainty transformation. The confidence level for a consensus set can be measured by computing the probability that a consensus set of size  $|C_r|$  or higher occurs by accident in the given sets of features. This measure enhances the robustness of the proposed matching method: by selecting only those sets for which that probability is



Fig. 4. Frame (b) is taken a few *ms* before frame (a) in an example sequence depicting a real life traffic situation obtained from a moving car.

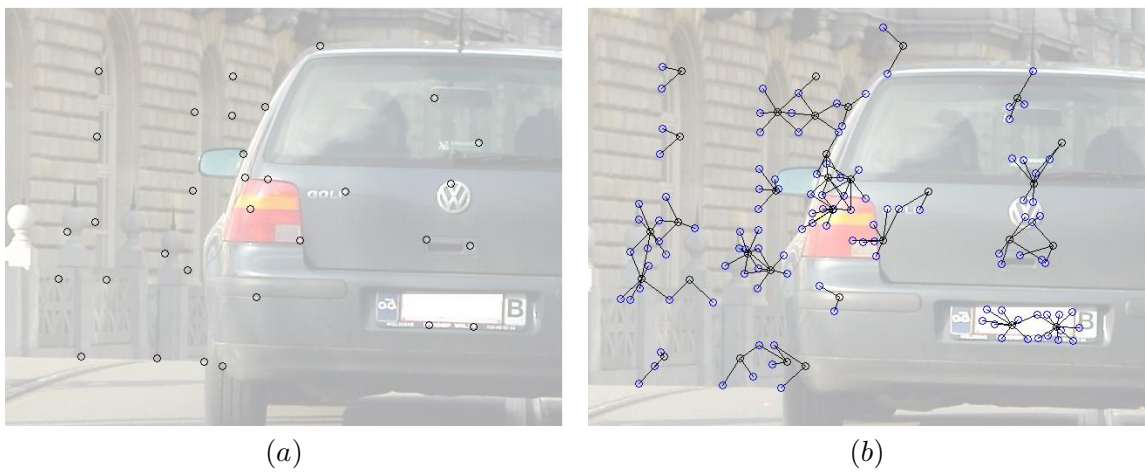


Fig. 5. The features in frame (a) are detected using a Harris corner detector. The features in the next image of the sequence (b) are then detected in a region of interest, the location of which is determined by the position of the reference features.

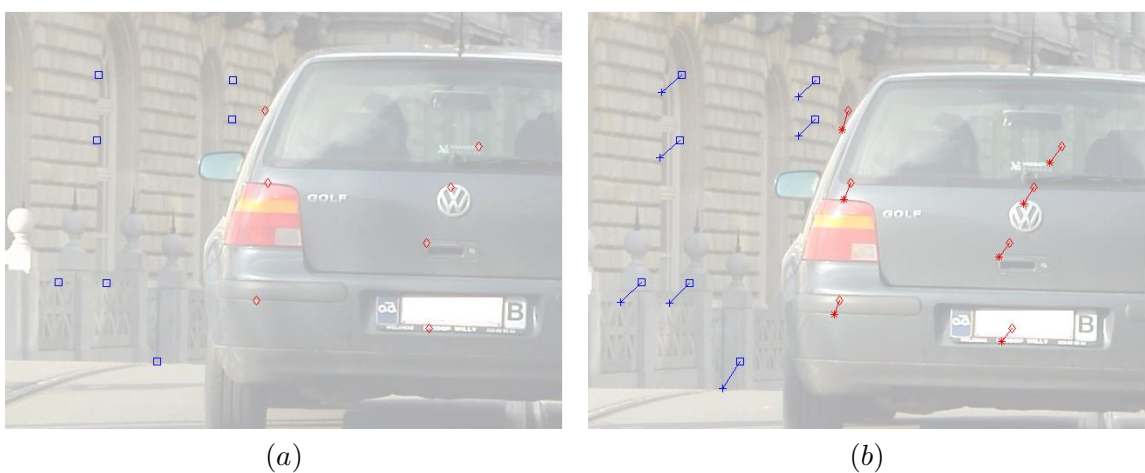


Fig. 6. Matched features in the reference frame (a) and the correspondence pairs in the next frame from the sequence (b). Two consensus sets for two different transformations are obtained and shown by different symbols.

sufficiently low, the chances for finding a correct correspondence set increase considerably. We can also determine which are the interesting combinations of two (or more) correspondence pairs: the confidence level and the size of the consensus set for the combination gives an indication about how reliable this correspondence set can be.

#### REFERENCES

- [1] C. Schmid A. Zisserman J. Matas F. Schaffalitzky T. Kadir K. Mikolajczyk, T. Tuytelaars and L. Van Gool, "A comparison of affine region detectors," *Accepted for International Journal of Computer Vision*, 2005.
- [2] R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, Cambridge University Press, Cambridge, 2003.
- [3] K. Teelen and P. Veelaert, "Uncertainty of affine transformations in digital images," *Proceedings of ACIVS 2004*, pp. 23–30, 2004.
- [4] K. Teelen and P. Veelaert, *Image registration using uncertainty transformations*, vol. 3708 of *Lecture Notes in Computer Science*, pp. 348–355, Springer, 2005.
- [5] P. Veelaert, *Collinearity and weak collinearity in the digital plane*, vol. 2243 of *Lecture Notes in Computer Science*, pp. 434–447, Springer, 2003.
- [6] W. Forstner, *Uncertainty and Projective Geometry*, Handbook of Computational Geometry for Pattern Recognition, Computer Vision, Neurocomputing and Robotics. To appear, Springer, 2004.
- [7] C. Harris and M. Stephens, "A combined corner and edge detector," *Proc. Alvey Vision Conf., Univ. Manchester*, pp. 147–151, 1988.