

# On image quality enhancement in Photoacoustic image reconstruction by motion compensation

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*Abstract*— Photoacoustic (PA) imaging is a relatively new noninvasive medical imaging modality. It is a technique which is harmless for the human body and uses pulsed optical energy. The process is based on the absorption of the pulse of optical energy by an object leading to local temperature increases. This temperature distribution leads to the generation of pressure waves, which in turn can be measured outside of the object. The measured acoustical signals at the boundary of the object can be used to reconstruct the original energy absorption distribution.

Reconstruction of the absorption distribution involves back projecting of the ultrasound measurements in some spatial map. Unexpected movements of the object can distort this reconstruction and result in blurred images. The aim of this work is to research a technique that corrects for these unexpected and a priori unknown movements.

The approach is based on tracking known landmarks in the received signal. By assuming rigidness of the object under consideration, the tracking of landmarks in the PA signal can be used to estimate small translations of the object during the signal acquisition.

*Keywords*— Photoacoustic imaging, motion correction, state estimation

## I. INTRODUCTION

In photoacoustic imaging, pulsed optical energy is delivered to an object with an unknown optical absorption distribution. The absorption of optical energy, which leads to a local increase in temperature, generates pressure waves which can be measured outside of the object. The relation between the optical absorption distribution and the generated pressure waves can be deduced from the governing wave equation [1]:

$$\nabla^2 p(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 p(\mathbf{r}, t)}{\partial t^2} = -A(\mathbf{r}) \frac{\beta}{C} \frac{dI(t)}{dt} \quad (1)$$

where  $c$  is the speed of sound,  $\beta$  is the volume thermal expansion coefficient and  $C$  is the specific heat.  $A(\mathbf{r})$  is the unknown absorption distribution and  $I(t)$  is the illumination function. A solution to this wave equation can be found by the use of Green's function. In the case of illumination with a pulse of light this

becomes

$$p(\mathbf{r}, t) = \frac{d}{dt} \frac{\beta}{4\pi C} \iiint \frac{A(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \delta\left(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right) d\mathbf{r}' \quad (2)$$

where  $\delta(t)$  is the dirac delta function. These generated pressure waves can be measured with a transducer outside of the object. The measured pressure by a transducer is influenced by its transfer function and results in a measure of

$$\tilde{p}(\mathbf{r}, t) = p(\mathbf{r}, t) * h(t) \quad (3)$$

where  $*$  indicates convolution and  $h(t)$  is the impulse response of the ultrasound transducer. The signal  $\tilde{p}(\mathbf{r}, t)$  can be measured with an ultrasound transducer. Measurements can thus be taken at different positions and at different points in time.

In our approach we will take measurements at  $n$  different positions  $\{\mathbf{r}_1, \dots, \mathbf{r}_n\}$ , which describe a circle in a plane according to

$$\mathbf{r}_i = r \left[ \cos\left(\frac{i-1}{n}2\pi\right), \sin\left(\frac{i-1}{n}2\pi\right) \right]^T \quad (4)$$

where  $r$  is the radius of this circle. A measured signal at a position  $\mathbf{r}_i$  consists of samples at distinct time points

$$\mathbf{s}_i = \begin{bmatrix} \tilde{p}(\mathbf{r}_i, t_0^{(i)}) \\ \vdots \\ \tilde{p}(\mathbf{r}_i, t_m^{(i)}) \end{bmatrix} \quad (5)$$

where the relation between the time points is given as  $t_{j+1}^{(i)} = t_j^{(i)} + T$  with  $T$  the period of the sampling frequency. Because measurements at different positions are not taken at the same time (i.e.  $t_0^{(i)} \neq t_0^{(i+1)}$ ), we have to take into account the dynamics of the system. The most important aspect here is the uncontrolled/unwanted movement of the object during measurement acquisition.

## II. PROBLEM FORMULATION

Our problem can be defined as estimating the unknown object motion during measurement acquisition

and to use the estimated object motion in the image reconstruction process. In order to solve this problem we will formulate and test a method based on tracking landmarks of the object during image acquisition.

We assume that the structure of the object is rigid. The unwanted movement of the object during measurement acquisition can thus be described as a rigid transformation over time. In our setup we take measurements in a 2-dimensional plane, and consequently we can only reconstruct 2-dimensional images of the imaged structure. We will describe the rigid transformation of the object at each measurement step  $i$  with the state vector  $\mathbf{x}_i$ . For consecutive measurement steps, the changes in the transformation are small. The dynamics of the system can be captured as

$$\mathbf{x}_{i+1} = f(\mathbf{x}_i, \mathbf{v}_i) \quad (6)$$

where  $\mathbf{v}_i$  is a random variable. In our case we have

$$f(\mathbf{x}_i, \mathbf{v}_i) = \mathbf{x}_i + \mathbf{v}_i \quad (7)$$

because we assume that the movement is smooth, and restricted by the variance of the random variable  $\mathbf{v}_i$ .

The landmarks in the object will help us to estimate the evolving state vector. Observations of these landmarks will be used to decrease the uncertainty of the estimated state vector. A relation between observations and the current state  $\mathbf{x}_i$  can in general be described as

$$\mathbf{z}_i = h_i(\mathbf{x}_i, \boldsymbol{\varphi}, \mathbf{w}_i) \quad (8)$$

where  $\mathbf{z}_i$  is a vector of observations,  $\boldsymbol{\varphi}$  are parameters that influence the measurement function and  $\mathbf{w}_i$  is a random variable describing the uncertainty in the observed measurements. When tracking landmarks, the observations can be the time of flight values of the individual landmarks in the object. The exact values of the parameters  $\boldsymbol{\varphi}$  are important for a good estimation of the state vector  $\mathbf{x}_i$  but can sometimes still be uncertain known a-priori. The a-priori knowledge available on the parameters can be quantified in a probability density function (pdf)  $p(\boldsymbol{\varphi})$ .

What we are interested in after all are the values of  $\mathbf{x}_{1:n}$  and  $\boldsymbol{\varphi}$  which maximize the pdf

$$p(\mathbf{x}_{1:n}, \boldsymbol{\varphi} | \mathbf{z}_{1:n}) \quad (9)$$

representing the uncertainty on  $\mathbf{x}_{1:n}$  and  $\boldsymbol{\varphi}$ , given the observations  $\mathbf{z}_{1:n}$ . The problem here is that both the state vector and the parameters have to be estimated. We formulated this as an iterative optimization process. By using Bayes theorem and dropping non informative terms from the pdf, equation (9) can be

written as

$$\frac{p(\mathbf{z}_1 | \mathbf{x}_1, \boldsymbol{\varphi}) p(\mathbf{x}_1) p(\boldsymbol{\varphi})}{p(\mathbf{z}_1)} \prod_{i=2}^n \frac{p(\mathbf{z}_i | \mathbf{x}_i, \boldsymbol{\varphi}) p(\mathbf{x}_i | \mathbf{x}_{i-1})}{p(\mathbf{z}_i | \mathbf{z}_{i-1})} \quad (10)$$

using the relations (6) and (8).

### III. SOLUTION USING LINEARIZATION

We assume that the random variables  $\mathbf{v}_i$  and  $\mathbf{w}_i$ , representing the uncertainty in the state dynamics and the observations respectively, are Gaussian distributed. If now both the state dynamics and the measurement function were linear, optimization of the posterior pdf (9) would result in finding the minimum of a quadratic function. This minimum is easily found and thus we try to base our solution to the problem on linearization of the measurement function.

The quadratic function that we now have to optimize consists of a part representing the prior knowledge of the unknown variables and a part representing knowledge on observations of the unknown variables. This function is derived from (9) which is, due to the linearization, a product of Gaussians. By taking the logarithm and removing terms which are not dependent on the unknown variables, we end up with the sum of squared Mahalanobis distances of each of the individual pdfs in (10).

$$g(\mathbf{x}_{1:n}, \boldsymbol{\varphi}) = g_{\text{obs}}(\mathbf{x}_{1:n}, \boldsymbol{\varphi}) + g_{\text{prior}}(\mathbf{x}_{1:n}, \boldsymbol{\varphi}) \quad (11)$$

This quadratic function can be written as (after substituting  $\mathbf{x} = [\mathbf{x}_{1:n}^T, \boldsymbol{\varphi}^T]^T$ )

$$g(\mathbf{x}) = g_0 + \mathbf{G}^T \mathbf{x} + \mathbf{x}^T H \mathbf{x} \quad (12)$$

where  $H$  is the Hessian matrix of  $g$ . This function has a unique minimum when the Hessian matrix is positive definite, which is guaranteed by including the prior knowledge on the unknown variables. The minimum can subsequently be found by setting the gradient of  $g$  to zero and solving the linear system of equations

$$\mathbf{G} + 2H\mathbf{x} = 0 \Leftrightarrow \mathbf{x} = -2H^{-1}\mathbf{G} \quad (13)$$

In combination with the linearization this is an iterative optimization procedure, which is also known as Newton's method [2]. The solution to the linear system of equations is used as the linearization point in the next iteration. The optimization procedure will be initialized with the linearization point set at the expectation value of the unknown variables given the a-priori information

$$\mathbf{x}_i^{(0)} = \mathbb{E}(\mathbf{x}_i) \quad (14)$$

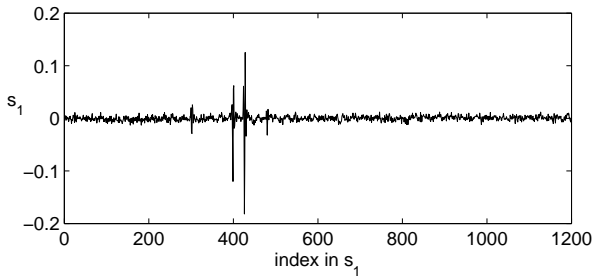


Fig. 1: Measured pressure signal  $\mathbf{s}_1$

$$\varphi^{(0)} = \mathbb{E}(\varphi) \quad (15)$$

#### IV. IMPLEMENTATION OF THE SOLUTION

In this section the implementation of the motion correction algorithm will be explained. To illustrate the individual steps in the algorithm, we show the results of each of the steps applied to measurements obtained from experiments on a phantom object.

The phantom object used here was a structure containing four landmarks, where each landmark is in fact a small hair with a diameter of  $150 \mu\text{m}$ . A series of measurements is made by rotating an ultrasound sensor around this structure, describing a full circle in 120 steps (i.e. an increase in angle by  $3^\circ$  each time). The phantom is illuminated with a laser pulse and the generated pressure wave is subsequently measured at these 120 positions in a circle around the phantom, which results in  $n = 120$  vectors of measurements  $\{\mathbf{s}_i\}_{i=1}^n$ . The measurements are sampled at a frequency of 20 MHz and the speed of sound  $c$  in the medium is assumed to be  $1500 \text{ m/s}$ . A single measurement vector  $\mathbf{s}_1$  is displayed in Fig. 1. The cropped version of the whole set of measurement vectors (vertically stacked) is displayed in Fig. 2a.

##### A. Signature of the landmark

Landmarks in the imaged object will be visible in the measured signal as a distinctive waveform of a certain length. We call this distinctive waveform the signature of the landmark and we have to determine this signature first, so that it can be used to extract time of flight (tof) measurements of the landmarks from the measured signal. Our phantom object contains only the four landmarks, so it should be easy to detect the landmark signature by looking at those parts of the measurement vectors which show a waveform above noise level. To see which parts of each the measurement vector  $\mathbf{s}_i$  are above noise level, we calculated the envelope of the signal by taking the absolute value its

analytic signal. The envelope of the phantom object measurements is displayed in Fig. 2b. The maxima in this envelope signal can subsequently be used to find potential realizations of the landmark signature in the neighborhood of those maxima. However, due to noise in the measurements, not all maxima found in the envelope signal are in reality caused by a landmark. To be robust to these outliers, we can use the fact that for one landmark the tof trajectory is periodic and approximate sine shaped (this can be observed in Fig. 2b as well). The maxima which are then too far away from one of the found sinoids can be marked as outliers. We can thus parametrize the tof trajectory as a sine function with a known period, but unknown offset, amplitude and phase. The offset however is the same for each of the sine functions generated by the different landmarks, and can be determined by taking the mean value of all tofs on which maxima have been found in the envelope signal. This leaves us with two parameters, amplitude and phase, for each of the four sine waves (because we have four landmarks in our example).

The problem that has to be solved now is the following, determine the most likely set of parameters that would generate an envelope image as we have observed it. The likelihood of a certain parameter vector can be evaluated by a Hough transform from the image to our parameter domain as

$$p(A, \theta) \propto \iint I(t, i) \delta \left( t - A \sin \left( \theta + i \frac{2\pi}{120} \right) \right) dt di \quad (16)$$

where  $I(t, i)$  is the envelope image as displayed in Fig. 2b. Because we are not interested in the actual intensities of the produced sinoids, we first preprocess the image  $I(t, i)$  so that only the positions of maxima in the envelope are of influence and the relative intensity of the maxima are discarded. This preprocessing is done by using the tofs of the estimated maxima to generate a new image, where we use a Gaussian kernel to model some of the uncertainty involved in the estimation of the maximum. The preprocessed image is then obtained as the sum of Gaussian kernels around the estimated maxima positions

$$I_{\text{pre}}(t, i) = \sum_{j=1}^4 e^{\frac{(z_{ij}-t)(z_{ij}-t)}{2\sigma^2}} \quad (17)$$

where the vector  $\mathbf{z}_i = [z_{i1}, \dots, z_{i4}]^T$  contains the tofs of the maxima found in the envelope of each signal  $\mathbf{s}_i$ . The preprocessed image obtained from the envelope image by using a kernel width of  $\sigma = 3$  is displayed

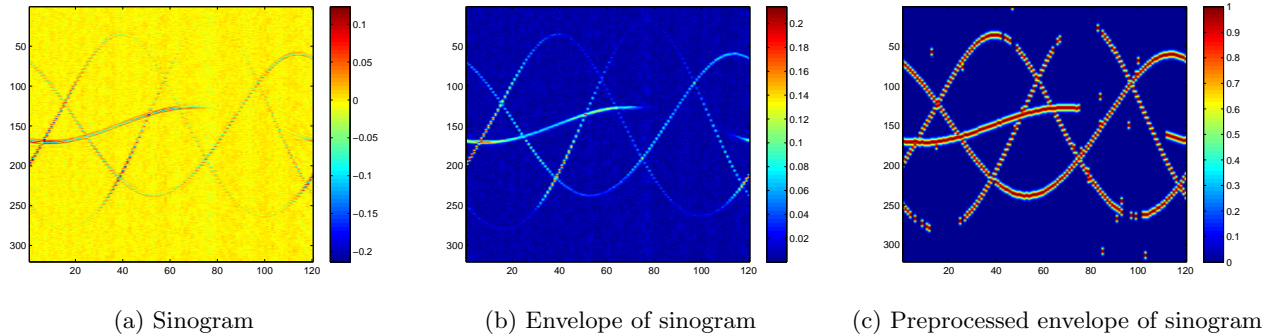


Fig. 2: Cropped measurements

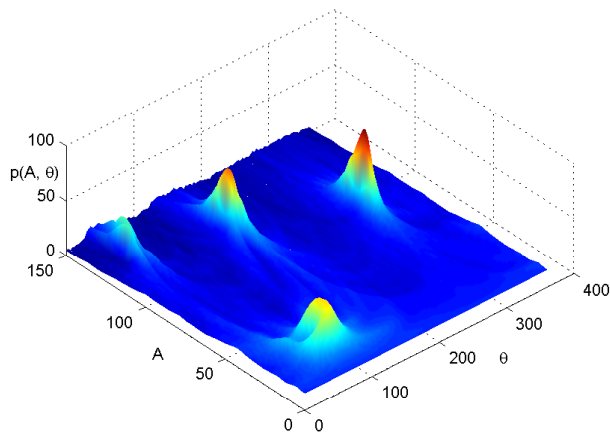


Fig. 3: Probability density function  $p(A, \theta)$

in Fig. 2c. The pdf  $p(A, \theta)$  can now be calculated by applying the Hough transform as given in (16) using the preprocessed image. The resulting pdf is displayed in Fig. 3. This pdf contains clearly four maxima, which are the parameters that generated the four sine waves. The four sets of parameters, being the four maxima in the pdf  $p(A, \theta)$ , can now be used to point out the outliers in the data.

We now know which parts of the measured signal, based on the positions of the estimated maxima, contain a realization of the landmark signature. These parts can be extracted from the signal and used for further estimation of the landmark signature. However, the exact location of the signature inside each extracted signal is not known. In order to align the extracted signals with respect to each other, we first increase the resolution by doing a cubic spline interpolation of the extracted signal and resample this interpolated signal at a higher sample rate. This allows us to

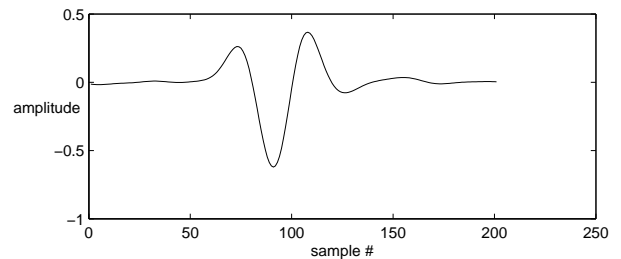


Fig. 4: Estimated landmark signature (upsampled with a factor of 10 by interpolation)

do more accurate alignment of the extracted signals. After observing the extracted signals, the alignment is based on the heuristic of aligning the zero crossing corresponding to the highest maximum to minimum difference in the signal.

It is possible that among the aligned signatures there are still outliers left, for example due to incorrect alignment or otherwise corrupted signals. These are removed by assuming that the aligned dataset is approximately Gaussian distributed. When any of the principal components in the dataset (based on eigenvector analysis of the covariance matrix) do not seem Gaussian distributed, the outliers are removed. The components are checked on their gaussianity by looking at the kurtosis of the dataset projected along the component. The finally estimated landmark signature is displayed in Fig. 4.

### B. Extraction of landmark measurements

Having defined the landmark signature, the next task is to extract the locations (tofs) of the landmark signatures from the measured pressure signals. This is done by correlation of the measured signal  $s_i$  with the landmark signature. The locations of the maxima

in the correlated signal are taken as the tof measurements. The number of correlation maxima that will be used is at most equal to the number of landmarks in the imaged object, but can be less if the value of a found maximum is below a certain threshold. This means that not all sensor measurements include an estimated tof for each landmark.

Here again we have to cluster the tof measurements according to their landmark source. To do this we will use a Hough transform, exactly as was described in the previous section on the landmark signature. This Hough transform gives the most likely parameters that generated the four sinoids. The tof measurements being closest to the estimated sinoid of the specified source (below some threshold distance to remove outliers) will be assigned to the corresponding source.

### C. Minimizing the cost function

With the tof measurements available and each measurement assigned to the correct landmark source, the next task is to minimize our cost function. This involves the use of a measurement function that relates the measurements to the unknown state vector of the transformations (object motion) and the parameter vector containing information about the measurement geometry to the tof measurements. We will model the state vector as the relative (in plane) translation of the object with respect to the first sensor reading

$$\mathbf{x}_i = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \quad (18)$$

The parameter vector in our case contains the positions of the four landmarks ( $\mathbf{p}^1, \dots, \mathbf{p}^4$ ), the radius of the circle on which the sensors are positioned ( $r$ ) and the synchronization time, being the difference between the time of firing of the laser pulse and time of receiving the first measurement sample ( $t_0$ ), all placed in one parameter vector

$$\boldsymbol{\varphi} = \begin{bmatrix} p_x^1 \\ p_y^1 \\ \vdots \\ p_x^4 \\ p_y^4 \\ r \\ t_0 \end{bmatrix} \quad (19)$$

The measurement function relates the state vector and the parameter vector to the tof measurements as

$$z_{ij} = h_{ij}(\mathbf{x}_i, \boldsymbol{\varphi}, \mathbf{w}_i) = \frac{\left\| \mathbf{p}_j + \mathbf{T}_i - r \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix} \right\|}{c} - t_0 + w_{ij} \quad (20)$$

where  $\theta_i$  is the rotation of the ultrasound sensor at the  $i^{\text{th}}$  measurement,  $c$  is the known speed of sound and  $\mathbf{w}_i$  is a Gaussian random vector with covariance  $P_{ww}$ . The subscript  $j$  indicates for which landmark the measurement function is used (since not all landmarks might have a tof measurement in each measurement  $\mathbf{s}_i$ ).

Our cost function is based on the linearization of the measurement equation and contains the sum of the Mahalanobis distances of each of the pdfs in (10). The measurement function can be linearized as

$$h_{ij}(\mathbf{x}_i, \boldsymbol{\varphi}, \mathbf{w}_i) \approx z_{ij}^{(l)} + H_x^{(l)} \mathbf{x}_i + H_\varphi^{(l)} \boldsymbol{\varphi} + \mathbf{w}_i \quad (21)$$

with

$$z_{ij}^{(l)} = h_{ij}(\mathbf{x}_i^{(l)}, \boldsymbol{\varphi}^{(l)}, 0) \quad (22)$$

where linearization is done about the point  $\mathbf{x}_i^{(l)}$  and  $\boldsymbol{\varphi}^{(l)}$  and  $H_x^{(l)}$  and  $H_\varphi^{(l)}$  are the appropriate Jacobian matrices. Using this linearization, the measurement likelihood can be written as

$$p(z_{ij}|\mathbf{x}_i, \boldsymbol{\varphi}) \sim \mathcal{N}(z_{ij}^{(l)} + H_x^{(l)} \mathbf{x}_i + H_\varphi^{(l)} \boldsymbol{\varphi}, P_{ww}) \quad (23)$$

The state transition function is already linear and has the simple form of

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \mathbf{v}_i \quad (24)$$

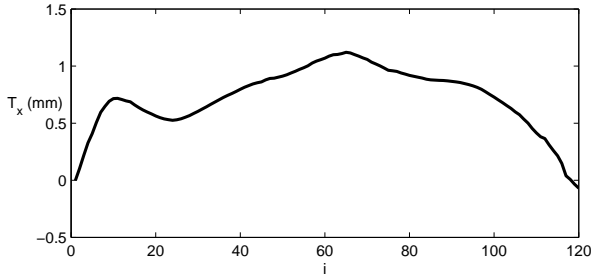
with  $\mathbf{v}_i$  a Gaussian random vector with covariance  $P_{vv}$ , so that we can write the transition pdf as

$$p(\mathbf{x}_{i+1}|\mathbf{x}_i) \sim \mathcal{N}(\mathbf{x}_i, P_{vv}) \quad (25)$$

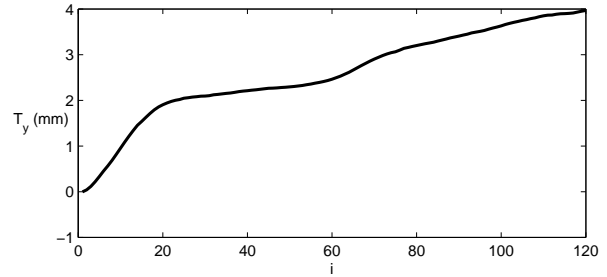
From these two pdfs, the Mahalanobis distances can easily be extracted and summed, resulting in the quadratic cost function. Iterative optimization of this cost function using the phantom object measurements converged quickly in 13 iterations to a solution. The evolving state vector  $\mathbf{x}_i$  in the final solution of the optimization is displayed in Fig. 5.

### D. Image reconstruction

The final task is to reconstruct the image from the obtained pressure measurements. For this we use a filtered backprojection algorithm, where we filter the



(a) Evolving state  $T_x$



(b) Evolving state  $T_y$

Fig. 5: Evolving state vector

signals prior to backprojection. We also take into account the transfer function of the ultrasound transducer, which distorts the measurements according to (3). The transducer function of our ultrasound sensor is in principle unknown, but we can estimate it from the observed landmark signature. Our landmark is a hair with a diameter of  $150 \mu m$ , of which we can predict the theoretically generated photoacoustic signal [3]. Using the theoretically predicted signal and the observed landmark signature, we calculated the impulse response of the transducer, and corrected the obtained ultrasound measurements for this.

Results of the image reconstruction are displayed in Fig. 6. Here we can clearly see the improvement that is obtained by correcting for the estimated object motion during the measurements acquisition. The uncorrected image clearly shows the motion, the reconstruction of the hairs does not result in four small dot, but instead we observe that the dots are somewhat distorted. This is shown more obvious in the zoomed in versions displayed in Fig. 7.

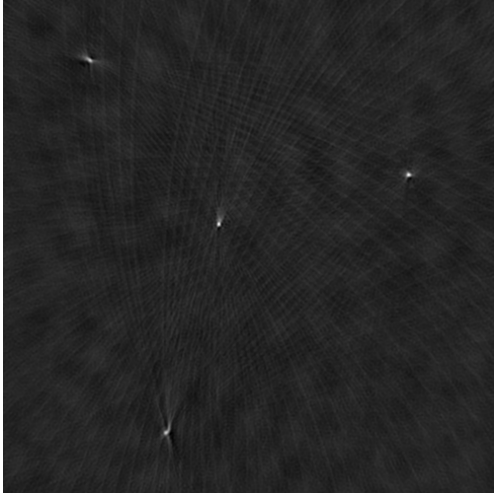
## V. CONCLUSION

In this paper we have demonstrated a method to correct for unwanted object motion during measurements acquisition, based on the tracking of landmarks in the object structure. Results have been presented and clearly show the improvement of the image quality. Because the method is based on the tracking of landmarks, these landmarks do need to exist/have to be attached to the object. How problematic this requirement is, is not clear yet, but for future work we are investigating a method that requires no landmarks in the structure. Instead of optimizing the likelihood function of the measurements, some sort of sharpness function will be used. The simultaneous estimation of geometry parameters and the transformation state

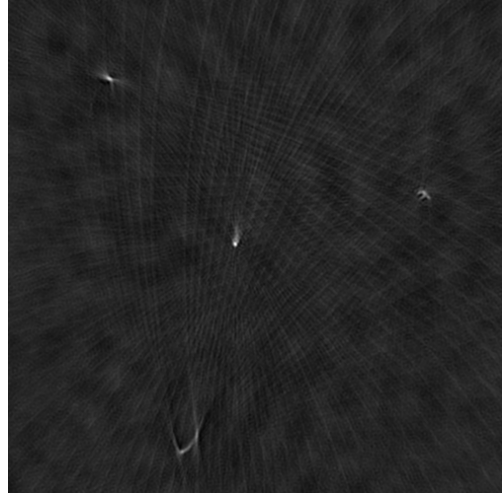
could be difficult in that case, but once the geometry parameters are fixed by calibration with a known object, they do not have to be estimated anymore.

## REFERENCES

- [1] R. R. A. Kruger, P. P. Liu, Y. Y. R. Fang, and C. C. R. Appledorn, "Photoacoustic ultrasound (paus)-reconstruction tomography," *Medical physics*, vol. 22, no. 10, pp. 1605-1609, 1995. 0094-2405.
- [2] R. Fletcher, *Practical methods of optimization*. John Wiley, 1987.
- [3] C. G. A. Hoelen and F. F. M. de Mul, "A new theoretical approach to photoacoustic signal generation," *J. Acoust. Soc. Am.*, vol. 106, no. 2, pp. 695-706, 1999.

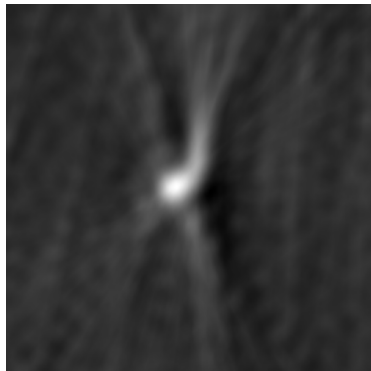


(a) Reconstructed image with motion correction

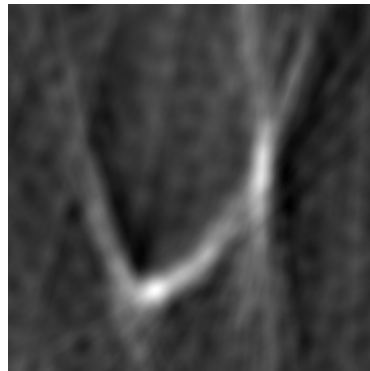


(b) Reconstructed image without motion correction

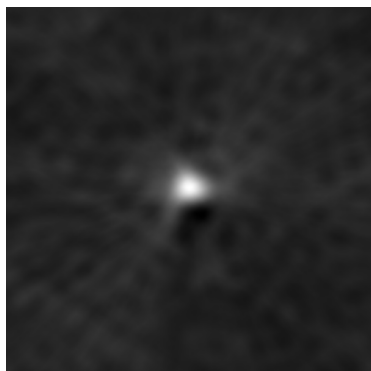
Fig. 6: Reconstructed image of the phantom object



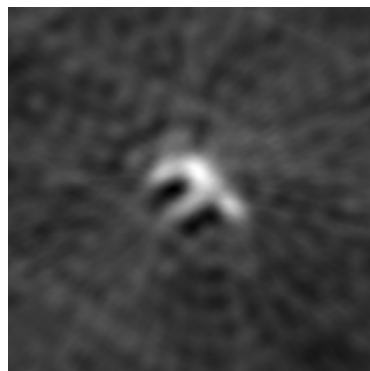
(a) Lowest hair, with motion correction



(b) Lowest hair, without motion correction



(c) Most right hair, with motion correction



(d) Most right hair, without motion correction

Fig. 7: Zoomed reconstruction two hairs in the phantom object (width = 2mm)