The State-Space Model for the Multi-Channel FM Separation System

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Abstract—**This paper is presented to separate multi-channel frequency modulation (FM) signals with additive white Gaussian noise (AWGN) by using state-space model. The FM system is used extensively in communication applications. When two or more FM signals are transmitted, even at high signal nosie ratio (SNR) conditions, the imperfect demodulated signals occur due to the multi channel interference. A demodulator for multi-channel interference FM systme is followed by constructing and combining the state-space models of phase-locked loop (PLL) and amplitude-locked loop (ALL), which has capability of demodulating FM signal transmissions and separating the dominant and other subdominants voice simultaneously. The analysis of the system is shown to separate the original modulating signal from mixed signals over AWGN channel without distortion, cross-talk or beat frequency.**

Keywords—**additive white Gaussian noise (AWGN); high signal nosie ratio (SNR); phase-locked loop (PLL); amplitude-locked loop (ALL)**

I. INTRODUCTION

The problems of separating multi-channel FM signals that are superimposed on each other arise in many signal processing for the communication algorithms and applications. Particularly, multi-channel signals separation is an interesting and important challenge, since it is useful for the separating competing method of the voice and denoising [1].

The main objective of this paper is to present state-space model for separation of multi-channel signals with linear matrix inequality (LMI) constraints. Thus, we treated discrete-time separation systems with additive noise in state-space model, proposed a dynamic finite impulse response (FIR) filter structure that consisted of the systems of PLL, ALL and low-pass filter [2], [3], [4],[5] and developed a framework in which the filter design was carried out by solving an LMI-constrained optimization problem. Here, these works did not contemplate the effect of channel distortion (fading, channel nonlinearities, etc.), which is often encountered in communications systems. However, to deal with the problem in the existence of linear time-invariant (LTI) channel distortion and additive noise, a mixed H_∞ filtering design scheme is supplied in the state space framework that yields a satisfactory reconstruction performance [6] [7].

The combined LMI approach was applied to improve the performance of any constant envelope multi-channel FM transmission system. We consider specifically the cochannel case and the results of the simulations are built multi-channel FM separation with AWGN and are proved the simulated performance.

II. SEPARATION MODEL DESCRIPTION

Figure 1 shows the separation model of cochannel case, where $j(k)$, $v(k)$ and $m_1(k)$ represent the modulated signal, an AWGN and the original signal, respectively. Then, we derive a state-space representation for the separating FIR filter transfer function in the receiver. The separation model of the filter model can be defined as [8], [9]

$$K(z) = k_0 + k_1 z^{-1} + \ldots + k_{L-1} z^{-(L-1)} \tag{1}$$

where $L-1$ and $k_0, k_1, \ldots, k_{L-1}$ are the filter orders and filter coefficients, respectively. It is represented by

$$w(k+1) = A_e w(k) + B_e y(k)$$
$$\hat{z}(k) = C_e w(k) + k_0 y(k) \quad (2)$$

where $w(k)$ is the state vector, $y(k)$ is the received input, $\hat{z}(k)$ is the estimated output of the separating FIR filter, $A_e = \begin{bmatrix} 0 & 0 \\ I_{(L-2)\cdot(L-2)} & 0 \end{bmatrix}$, $B_e = \begin{bmatrix} 1 \\ 0_{(L-2)\cdot 1} \end{bmatrix}$, $C_e = \begin{bmatrix} k_1 & \cdots & k_{L-1} \end{bmatrix}$.

Assume a channel model for the communication transmission whose impulse response is described by

$$C(z) = 1 + 0.33562z^{-1} + 4.6276z^{-2} - 0.14487z^{-3} + 1.6837z^{-4} \quad (3)$$

The channel contaminated with the noise at the output is described by the following state-space structure

$$s(k+1) = A_c s(k) + B_c u(k)$$
$$y(k) = C_c s(k) + D_c u(k) \quad (4)$$

with

$$A_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; B_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C_c = \begin{bmatrix} 0.33562 & 4.6276 & -0.14487 & 1.6837 \end{bmatrix}$$

and

$$D_c = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

where $s(k)$ is the state vector, $u(k) = [j(k) \quad v(k)]^T$ is the transmitted vector and superscript "T" denotes matrix transposition, $y(k)$ is the received input. The frequency response of $C(z)$ is illustrated for 5_length FIR channel in Figure 2, and it was seen how this channel distorted transmitted signals in magnitude and phase.

Consider the delay operator $R(z) = z^{-d} I$ is expressed by

$$r(k+1) = A_d r(k) + B_d u(k)$$
$$z(k) = C_d r(k) \quad (5)$$

where $d = 2$, $r(k)$ is the state vector, $A_d = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $B_d = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$, $C_d = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $\boldsymbol{I}$ denotes the identity matrix with appropriate dimension.

From the above preliminary, we acquired the state-space model for the reconstruction error as

$$e(k) = z(k) - \hat{z}(k) \quad (6)$$

The overall system for the separation system can be written as

$$x(k+1) = Ax(k) + Bu(k)$$
$$e(k) = Cx(k) + Du(k) \quad (7)$$

where $A = \begin{bmatrix} A_c & 0 & 0 \\ 0 & A_d & 0 \\ B_e C_c & 0 & A_e \end{bmatrix}$, $B = \begin{bmatrix} B_c & B_d & B_e D_c \end{bmatrix}^T$, $C = \begin{bmatrix} -k_0 C_c & C_d & -C_e \end{bmatrix}^T$, $D = \begin{bmatrix} -k_0 D_c \end{bmatrix}$, and $x(k) = \begin{bmatrix} s(k) & r(k) & w(k) \end{bmatrix}^T$

Therefore, the representation of error transfer function is specified as

$$T(z) = C(zI - A)^{-1} B + D \quad (8)$$

Our objective is to reconstruct $u(k)$ from the received signal $y(k)$, which is corrupted by additive noise, and minimize the reconstruction error $e(k)$ according to certain filtering criterion [7].

III. H_∞ FILTERING

The major results are based on the discrete-time algebraic Riccati Inequality (DARI). In this section, we show how to cast the control problem into the LMI form and introduce the following intermediate technical results for the discrete algebraic Riccati equation.

Definition: Consider the discrete algebraic Riccati equation

$$A^T PA - P + [(A^T PB + M^T)(R - B^T PB)^{-1} (B^T PA + M)] + Q = 0 \quad (9)$$

where A, B, Q, R and M are real matrices of appropriate dimensions, and Q and R are symmetric matrices. A real symmetric matrix P is said to be stabilizing solution to Eq. (9) if P satisfies Eq. (9) and matrix $\overline{A} = A + B(R - B^T PB)^{-1}(B^T PA + M)$ stable.

Lemma: [10] Suppose A is a stable matrix and that there exists a matrix $\hat{P} = \hat{P}^T$ such that $R - B^T PB > 0$ and

$$A^T \hat{P}A - \hat{P} + [(A^T \hat{P}B + M^T)(R - B^T \hat{P}B)^{-1} (B^T \hat{P}A + M)] + \hat{Q} = 0 \quad (10)$$

where $A, B, \hat{Q}, R$ and M are matrices with appropriate dimensions with $\hat{Q}$ and R being symmetric and $R > 0$. Then, for any symmetric matrix Q such that $0 \le Q \le \hat{Q}$ the equation

$$A^T PA - P + [(A^T PB + M^T)(R - B^T PB)^{-1} (B^T PA + M)] + Q = 0 \quad (11)$$

have an unique strong solution $P = P^T$ satisfying $R - B^T PB > 0$. Furthermore we have that $0 \le P \le \hat{P}$.

Now, we consider the discrete-time case. An instrumental result to convert the H_∞ constraints into LMI is the bounded real lemma. This lemma establishes the equivalence between the following statements:

Theorem: Consider a discrete-time transfer function $T(z)$. The following statements are equivalent:

(i). $\left\| C(zI - A)^{-1}B + D \right\|_\infty < \gamma$ and A is stable in the discrete-time sense, where the γ value is a tolerance level.

(ii). There exists a stabilizing solution $\hat{P} = \hat{P}^T \ge 0$ to Riccati equation

$$A^T \hat{P}A - \hat{P} + \gamma^{-2}(A^T \hat{P}B + C^T D)[I - \gamma^{-2}(D^T D^{-1} + B^T \hat{P}B)]^{-1}(B^T \hat{P}A + D^T C) + C^T C = 0 \quad (12)$$

such that $I - \gamma^{-2}(D^T D + B^T \hat{P}B) > 0$

(iii). There exists a matrix $P = P^T \ge 0$ satisfying

$$A^T PA - \hat{P} + \gamma^{-2}(A^T PB + C^T D)[I - \gamma^{-2}(D^T D^{-1} + B^T PB)]^{-1}(B^T PA + D^T C) + C^T C < 0 \quad (13)$$

and such that $I - \gamma^{-2}(D^T D + B^T PB) > 0$. Moreover, $\hat{P} < P$.

(iv). There exists a solution $P > 0$ to the LMI:

$$\begin{bmatrix} A^T PA - P & A^T PB & C^T \\ B^T PA & B^T PB - \gamma^2 I & D^T \\ C & D & -I \end{bmatrix} < 0 \quad (14)$$

IV. SIMULATION

Consider the separating FIR filter

$$S(z) = 0.1322\left\{\left[\frac{(2m)\vartheta^3 + (1-m)\vartheta^2 + (1-m)\vartheta + (-1+m)}{\vartheta^3}\right] + \left[\frac{(2-2m)\vartheta^2 + (1-m)\vartheta + (-1+m)}{\vartheta^2}\right]z^{-1} + \left[\frac{(-1+m)}{\vartheta}\right]z^{-2}\right\} \quad (15)$$

where $\vartheta = \cos\omega_d$, m is the interference to carrier ratio and ω_d instantaneous difference frequency.

Taking (15) into (2), we obtain

$$A_e = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad (16)$$

$$B_e = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (17)$$

$$C_e = \begin{bmatrix} \dfrac{(2-2m)\vartheta^2 + (1-m)\vartheta + (-1+m)}{\vartheta^2} \\ \dfrac{(-1+m)}{\vartheta} \end{bmatrix}^T \quad (18)$$

$$k_0 = \frac{(2m)\vartheta^3 + (1-m)\vartheta^2 + (1-m)\vartheta + (-1+m)}{\vartheta^3} \quad (19)$$

Using (16), (17), (18) and (19) into (7), we obtain

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.33562 & 4.6276 & -0.14487 & 1.6837 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} -0.0693 \\ -0.9555 \\ 0.0299 \\ -0.3476 \\ 0 \\ 1 \\ -0.0197 \\ 0.0271 \end{bmatrix}^T$$

$$D = \begin{bmatrix} -0.2065 & -0.2065 \end{bmatrix}$$

We found a feasible common ***P*** matrix is positive defined and conclude that the system is stable and the optimal γ values obtained is 0.577.

$$P = \begin{bmatrix} 3.0201 & -0.0033 & 0 & 0 & -3.0201 & 0.0033 & 0 & 0 \\ -0.0033 & 1.5140 & 0 & 0 & 0.0033 & -1.5140 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3.0201 & 0.0033 & 0 & 0 & 3.0201 & -0.0033 & 0 & 0 \\ 0.0033 & -1.5140 & 0 & 0 & -0.0033 & 1.5140 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To evaluate the reconstruction performance, we define the receiver output signal-to-noise ratio (SNR) as a relative measure of reconstruction error [7]

$$SNR_o := 10\log_{10} \frac{\sum_k m_1^{\,2}(k)}{\sum_k e^2(k)} \qquad (20)$$

while the receiver input SNR is defined as following

$$SNR_i := 10\log_{10} \frac{\sum_k j^2(k)}{\sum_k v^2(k)} \qquad (21)$$

The performance comparison in linear case is listed in the following table, and illustrated in Figure 3.

SNR_i	-20	-5	0	10	20
$SNR_o^{m=0.5}$	3.4740	5.4374	5.9527	6.1600	6.3348
$SNR_o^{m=0.9}$	2.5505	4.6307	4.9442	5.1986	5.2787

V. CONCLUSIONS

In this paper, we described a method of separating FIR filter of multi-channel FM systems and proposed a so-called H_∞ filter approach. The LMI approach takes advantage of a H_∞ filter in order to maintain the reconstruction performance. We formulated into an optimization problem with standard LMI. Simulation results show different *m* value to prove the separation system performance in LMI constraints.

From Figure 3, it can be found approximately 1dB difference for the input SNR between -20dB and 20dB. In the future, we also build and simulate the performance of index for the low SNR ratio of the channel noise in fading channel condition.

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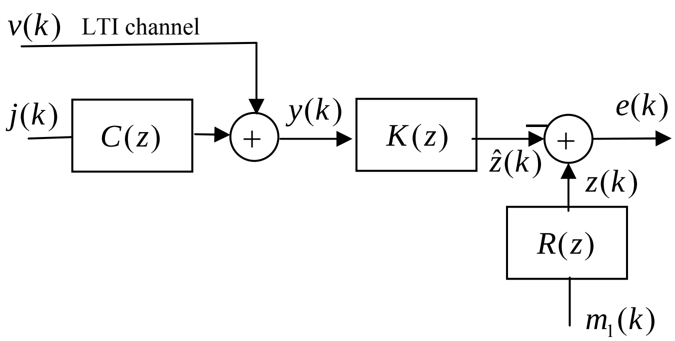

Figure 1 Separation model with linear time-invariant channel.

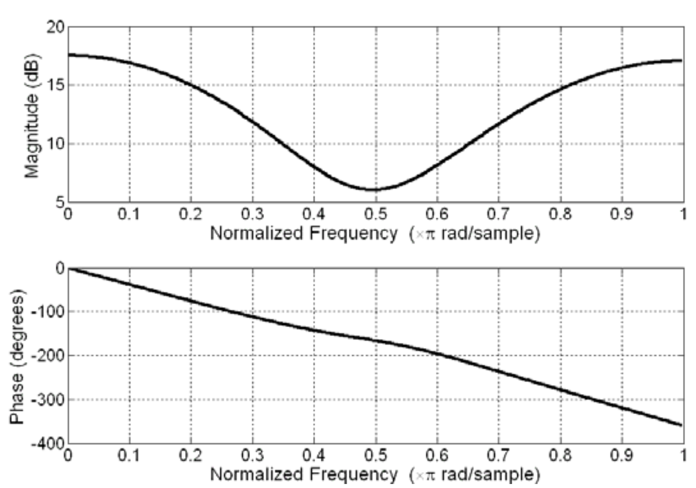

Figure 2 Frequency responses of the communication channel $C(z)$.

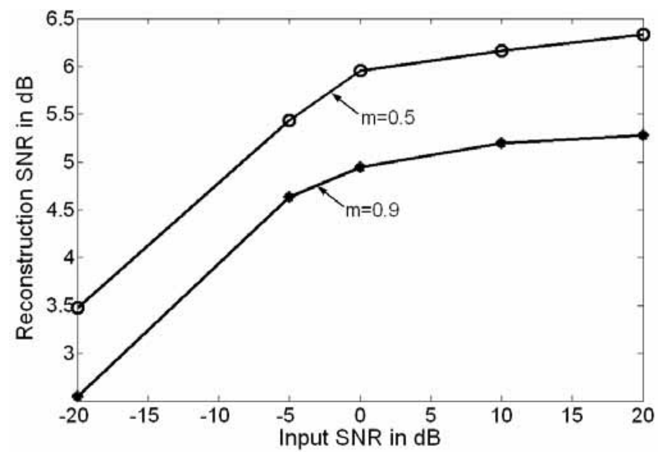

Figure 3 Reconstruction performance of the $m = 0.5$ “o” and $m = 0.9$ “*”.