Comparison of the orthodox theory of single electronics and the impulse circuit model for use in nanoelectronics

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Abstract— **The application of emerging nanoelectronic devices is only possible if a good circuit theory exists for designing nanoelectronic circuits. Basic to such a circuit theory is that it includes quantization of electrical charge. Also other quantum mechanical phenomena might be included, i.e., tunneling. In this paper two theories are compared that deal with nanoelectronic metallic single-electron tunneling devices. One is the orthodox theory of single electronics, which translates the quantization of charge into the quantization of the continuous spectrum of energy states associated with small metal islands. The other is the impulse circuit model, which translates the quantization of charge into the quantization of time during a tunnel event. Both theories will be described and compared for suitability of guiding the electronics engineer towards a circuit theory. The conclusion is that the impulse circuit model better fits the requirements for a good circuit theory.**

Keywords—**Single-electron tunneling devices, nanoelectronics, quantum devices, single-electron transistors and circuits, single-electron tunneling, hot electron.**

I. INTRODUCTION

For future electronics, nanoelectronics might be a breakthrough. New nanoelectronic devices may provide electronic circuits having very small basic devices and operation with very low supply power per device. Another motivation for nanoelectronics is the prospect to take up powerful circuit concepts from the past, e.g., threshold logic and multiple-valued logic, which former rejection resulted from missing an area efficient implementation with then existing technologies [3]. Together with the more novel computational concepts, as systolic arrays, artificial neural networks, and cellular automata nanoelectronics may tackle the interconnection problem that will soon become relevant in CMOS technology [1].

To understand the possible utilization of nanoelectronic devices, however, useful and competitive circuits have to be designed. New circuit ideas must be developed exploiting the quantum character, the small feature size, and the low-power operation of the new nanoelectronic devices. Structuring the design problem is essential for reducing the complexity [2]. And besides this, it is essential that a good circuit theory exists for designing nanoelectronic circuits.

The basic quantum mechanical phenomena that play a role in nanoelectronics are the quantization of charge, tunneling through potential barriers, and Coulomb blockade. The last being the suppression of tunneling in structures small enough to show tunneling of electrons.

In nature charge is quantized. Experiments show that the charge of any particle is always a multiple of e, e $= 1.6 \times 10^{-19}$ C. The nanoelectronic devices considered here, use explicitly this quantum nature. The circuit in which the nanoelectronic device is embedded, however, describes charge as being continuous in space. This means that we consider a discrete transfer of electrons through the tunneling junctions coexisting with continuous charge transport through the remainder of the circuit, the environment. Such a description causes two different time scales in the circuit. One involves the time within the discrete tunnel event takes place, the other is associated with the natural response of the environment.

If *many* electrons are tunneling it is possible to describe this tunneling with a tunnel current, and thus averaging the discrete nature of the electrons. In a circuit theory those devices can then be modeled as (nonlinear) resistors. The new phenomena of Coulomb blockade and Coulomb oscillations, however, are the result of tunneling of *single* electrons. Physically, a single tunnel event can best be understood as an electron sitting at *both* sides of the tunnel barrier *at the same time*, but the probability of finding the electron going from one side of the barrier to the other is decreasing on one side and increasing on the other during tunneling. As a consequence charge is moved inside the tunneling junction. But also time is involved. The period of interaction between the electron and the barrier is called the tunneling time, typically 10^{-15}s. The Coulomb blockade effect gives rise to a description of the metallic tunnel junction as a linear capacitor in the absence of tunneling.

Tunneling is explained by considering the wave nature of the electron, expressed by the De Broglie wave length $\lambda_{\mathrm{F}} = h/\sqrt{2mE_{\mathrm{F}}}$. Two metallic tunnel junctions in series form an island. To model the two junctions as capacitors (in case of Coulomb blockade) the island must be large compared with λ_{F}.

A few theories have been proposed to form the basis of a circuit theory for circuits including single-electron tunneling devices. The most widely applied orthodox theory of single electronics, see [4] and refs. therein, has a asymmetry in the description of the single-electron tunneling junction when excited with different sources. That is, "high-ohmic" and "low-ohmic" environments are treated differently. To a large extend this problem is due to the assumption of negligible tunneling time, a basic assumption in the orthodox theory.

In this paper a view assumptions are made:

- random background charges and initial charges on the islands are taken zero.
- behavior of the devices at a temperature of 0K.
- cotunneling is ignored.
- tunneling time is zero in the orthodox theories.
- tunneling time might be nonzero in the impulse circuit model.

These assumptions simplifies the equations, but does not really limit the description. The paper start with a precise definition of the single-electron charging and Coulomb energy to avoid confusion; a couple of different expressions exist in literature. After this, different approaches to explain the Coulomb blockade are critically reviewed. The single-electron charging approach was developed by Gabert [5], Devoret [6], Tucker [7], and later by Weis [8]. The electrostatic free-energy approach was pioneered by Likharev and Averin [9], [10], [4], and Hadley [11]. The impuls circuit model originates from Hoekstra and Klunder [12], [13], [14]. With applications in circuits in mind the free-energy approach using SIMON was taken by Lageweg [15], Zardalis [16], and Sulieman [17]. A more hybrid approach to circuit desing is described by Goodnick [18]. For an introduction to this area the interested reader is referred to recent reviews in this field [19], [20], [21], [22].

II. Energy Based Models

In this section two approaches to modeling Coulomb blockade in single-electron devices are reviewed and tested for their usefulness for designing nanoelectronic circuits that include these devices. First, energy based theories are discussed. In these approaches, using the single-electron charging energy, the quantization of charge leads to the quantization of the continuous spectrum of energy states associated with the capacitance of the small metal island. Together these approaches are often called the orthodox theory of single electronics. First we look at the approach based on the single-electron charging or Coulomb energy, and second the free-energy approach is briefly outlined. Both approaches are applied on a so-called double junction circuit, a circuit in which two single-electron tunneling junctions in series are excited by an ideal voltage source.

A. Single-Electron Charging Energy

Figure 1 shows a small metallic, initially electroneutral, island embedded in an insulating medium and surrounded by two metal electrodes. The insulator layer between the electrodes and the island is sufficiently small that the island can be charged and uncharged by electrons that tunnel to and from the island. The electrodes are connected through an electrical network N.

A first approach is to view the system as having a single capacitance $C_{\sum}$ in which the island reflects one electrode of this capacitor and the others form all together the other electrode. The overall charge of the system adds up to zero, that is, for the total island charge q_i holds:

$$q_i + q_1 - q_2 = 0, \tag{1}$$

where q_1 and $(-q_2)$ represents the charges on the electrodes.

The energy necessary to add an extra electron to the island onto which already n electrons have been tunneled can be called the *single-electron charging en-*

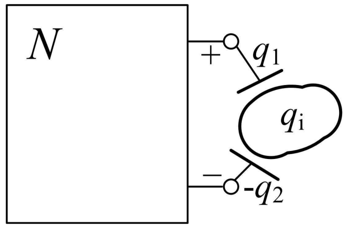

Fig. 1. An isolated metallic island with two electrodes that are connected by the network N

ergy $E_{\mathrm{ce}}(n)$:

$$E_{\mathrm{ce}}(n)=\frac{((n+1)\mathrm{e})^2}{2C_{\sum}}-\frac{(n\mathrm{e})^2}{2C_{\sum}}=\frac{\mathrm{e}^2}{2C_{\sum}}(2n+1). \qquad (2)$$

For initially uncharged islands the expression for the single-electron charging energy $E_{\mathrm{ce}}(n=0)$ is called the *Coulomb energy* E_{C}:

$$E_{\mathrm{C}}=\frac{\mathrm{e}^2}{2C_{\sum}}. \qquad (3)$$

It is the electrostatic energy barrier felt by the single electron moving onto or from an electrically neutral island.

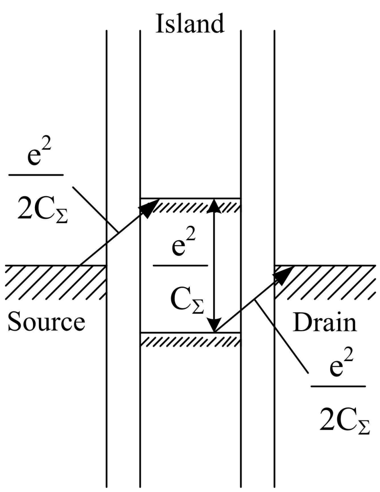

Fig. 2. The continuous spectrum of energy levels on the island is quantized and splitted due to single-electron charging phenomenon

For discussing the consequences of the single-electron charging energy think of one of the electrode to be a source and the other to be a drain. Assume that the two electrodes form tunnel junctions and that both tunnel junctions are identical. If we start from a condition in which the island is initially charge neutral, i.e., $n=0$, then it is clear from Equation 2 that a minimum amount of energy is necessary in either direction for tunneling to occur. The energy necessary to adding or removing an electron is the Coulomb energy $\mathrm{e}^2/(2C_{\sum})$. Essentially, the charging energy opens a gap of $\mathrm{e}^2/C_{\sum}$ in the continuous spectrum of energy states associated with the island, which forbids tunneling until this barrier can be surmounted when an energy source is applied during tunneling, see figure 2. In this idealized situation a Coulomb blockade exists if during tunneling not enough energy is supplied. For a current through the two junctions, energy for an electron to enter or to leave has to be provided first—the tunneling process corresponds then to the *correlated tunneling* of one electron towards the island and another from the island.

For the use of the Coulomb energy, the island size and therefore $C_{\sum}$ must be very small. For $C_{\sum}$ in the order of 10^{-18} F, E_{C} is of importance. In addition, to avoid thermal noise, E_{C} has to exceed E_{th}, with $E_{\mathrm{th}}=k_{\mathrm{B}}T$.

The application of this approach to single-electron circuits is illustrated by considering a simple circuit in which a small island is embedded in two electrodes denoted as source S and drain D, and energy is provided by a voltage source V_{DS}, see Figure 3. Now following [8], with increasing the bias voltage $V_{\mathrm{DS}}>0$, the electrostatic energy barrier for adding an electron from the source

$$\Delta E_{\mathrm{S}\to\mathrm{I}}=E_{\mathrm{C}}-\mathrm{e}\frac{C_{\mathrm{D}}}{C_{\sum}}V_{\mathrm{DS}} \qquad (4)$$

and the electrostatic barrier for an electron leaving the drain

$$\Delta E_{\mathrm{I}\to\mathrm{D}}=E_{\mathrm{C}}+\mathrm{e}\frac{C_{\mathrm{D}}}{C_{\sum}}V_{\mathrm{DS}}-\mathrm{e}V_{\mathrm{DS}} \qquad (5)$$

are reduced as a consequence of the applied voltage V_{DS}. Similar happens for $V_{\mathrm{DS}}<0$.

Filling in the value of the Coulomb energy, Equation 3, the suppression of a tunnel current, either $\Delta E_{\mathrm{S}\to\mathrm{I}}=0$ or $\Delta E_{\mathrm{I}\to\mathrm{D}}=0$, is finally overcome for

$$|V_{\mathrm{DS}}|>V_{\mathrm{DS}}^{\mathrm{cr}}\equiv\min\left(\frac{\mathrm{e}}{2C_{\mathrm{S}}};\frac{\mathrm{e}}{2C_{\mathrm{D}}}\right) \qquad (6)$$

and the drain-source current $|I_{\mathrm{DS}}|$ rises almost linear with increasing $|V_{\mathrm{DS}}|$. Equation 6, denoting the critical voltage, is accepted by most authors for predicting the Coulomb blockade in case of a double junction in series with an ideal voltage source.

But lets now think about this approach a little more closely. Therefore, we consider an initially uncharged island and we allow bounded currents only. In Equations 4 and 5 the energies provided by the bias source

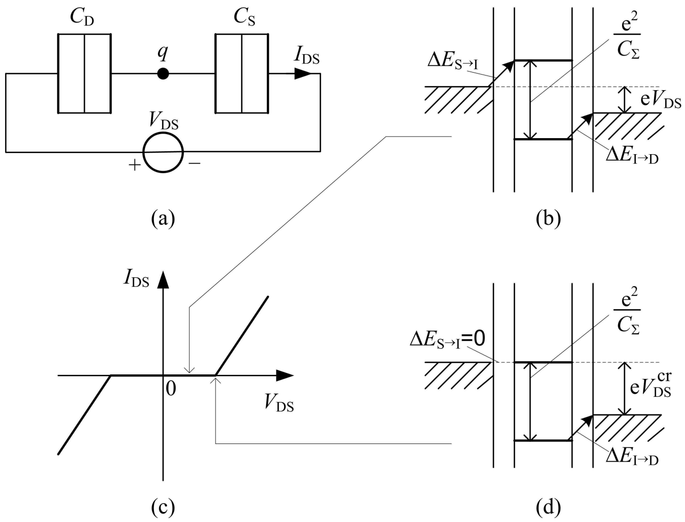

Fig. 3. Double junction circuit. The expected *VI*-characteristic and the location of the Fermi levels in two points of the characteristic.

$(C_D/C_\sum)eV_{DS}$ and $(1-C_D/C_\sum)eV_{DS}$ are the energies to charge the capacitances C_D and C_S with the fractions $Q_S = (C_D/C_\sum)e$ and $Q_D = (C_S/C_\sum)e$, respectively, *with a resistor.* Such a resistor has to dissipate half of the energy provided by the source and must have a non-zero value (however, *any* non-zero value will do).

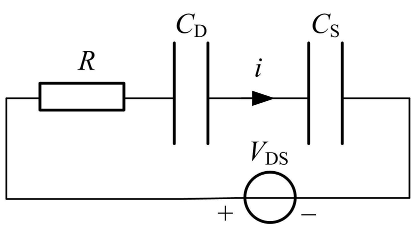

Fig. 4. Double junction circuit charged by an ideal voltage source and a resistor.

The resistor will cause a current in the circuit and, consequently, will determine the charges locally at the two different (opposite) sides of the metallic island, see Figure 4, and the voltage across the two junctions becomes a function of time. This immediately poses some questions. When does the electron tunnel, if it is tunneling at all? What is the influence of various value of R? And, moreover, *during* a tunnel event the source *does not* provide energy anymore, because the tunneling time is assumed to be zero, and both voltage and current are bounded. Until now we assumed that the source delivered energy *during* tunneling; is it possible that the source provides the energy *before* tunneling? A solution seems to be provided by the so-called free energy approach.

B. Electrostatic free-energy approach

In the free-energy approach, the tunneling of a single electron through a particular tunnel barrier or the presence of Coulomb blockade depends solely on the *reduction of free (electrostatic) energy of the system* as a result of tunneling. If there is a reduction of the system's free energy as a result of a single tunnel event, tunneling will occur (if ΔF is the change in the free energy then in case of reduction: $\Delta F < 0$). If there is no reduction of free energy possible as a result of a tunnel event, then the system is in Coulomb blockade. The evaluations are done at T $\approx$ 0 K, ignores possible energy quantization inside the conducting island, the tunneling time is assumed to be negligibly small, and cotunneling is not taken into account.

In general, the change of the free energy is defined as the change of the energy, w_{se}, stored on the (junction) capacitances minus the energy, w_s, delivered by the sources during a tunnel event. In terms of the change in free energy ΔF: If we define F_{before} and F_{after} as the free energy before and after the tunnel event, respectively, and $w_{se|before}$ and $w_{se|after}$ as the stored energy on the (junction) capacitances before and after the tunnel event, then $\Delta F = (F_{after} - F_{before})$, $\Delta w_{se} = (w_{se|after} - w_{se|before})$, and:

$$\Delta F = \Delta w_{se} - w_s. \tag{7}$$

Tunneling is possible if:

$$\Delta F < 0, \tag{8}$$

and the criterion for the system to be in Coulomb blockade is:

$$\Delta F \geq 0. \tag{9}$$

To describe the tunnel(ing) junction TJ as a circuit element, the free-energy approach uses the element's response to ideal voltage sources and the element's response to ideal current sources, possible dissipation in resistors cannot be considered in the approach.

To illustrate the approach we consider the double junction excited by an ideal voltage source (called the global view or "low-ohmic environment") again. First the most important formulas are derived and, second, the expressions for $v^{cr}(t)$, the voltage above which tunneling starts, in the special case that an electron tunnels through one of the junctions will be given. We consider the double-junction structure as illustrated in Figure 5 showing the most important parameters. Now, the goal is to obtain expressions for

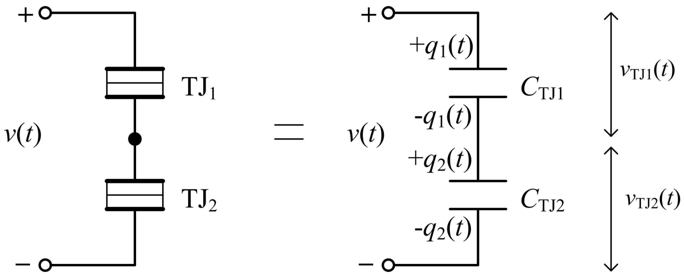

Fig. 5. Equivalent subcircuit for two tunnel junctions in Coulomb blockade.

w_s, the energy delivered by the source, and w_se, the energy stored on the junctions, for this circuit and to use Equation 8 to find the conditions for tunneling.

The following equations hold:

$$v(t) = v_\mathrm{TJ1}(t) + v_\mathrm{TJ2}(t). \tag{10}$$

and

$$q_\mathrm{i}(t) = -q_1(t) + q_2(t). \tag{11}$$

Modeling the junction capacitance as a linear time-invariant capacitor and using its definition, $q_k(t) = C_{\mathrm{TJ}k} v_{\mathrm{TJ}k}(t)$ for each capacitor, $k = 1, 2$, we obtain:

$$\begin{cases} v(t) & = & v_\mathrm{TJ1}(t) + v_\mathrm{TJ2}(t) \\ q_\mathrm{i}(t) & = & -C_\mathrm{TJ1} v_\mathrm{TJ1}(t) + C_\mathrm{TJ2} v_\mathrm{TJ2}(t). \end{cases}$$

Solving these two equations with two unknowns, we obtain:

$$v_\mathrm{TJ1}(t) = \frac{C_\mathrm{TJ2}}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} v(t) - \frac{1}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} q_\mathrm{i}(t), \tag{12}$$

$$v_\mathrm{TJ2}(t) = \frac{C_\mathrm{TJ1}}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} v(t) + \frac{1}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} q_\mathrm{i}(t). \tag{13}$$

We also obtain, using the capacitor definition:

$$q_1(t) = \frac{C_\mathrm{TJ1} C_\mathrm{TJ2}}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} v(t) - \frac{C_\mathrm{TJ1}}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} q_\mathrm{i}(t), \tag{14}$$

$$q_2(t) = \frac{C_\mathrm{TJ1} C_\mathrm{TJ2}}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} v(t) + \frac{C_\mathrm{TJ2}}{C_\mathrm{TJ1} + C_\mathrm{TJ2}} q_\mathrm{i}(t). \tag{15}$$

In *steady state* ($v(t) = v_\mathrm{s}$), we can now immediately obtain expressions for the stored energy in the junctions 1 and 2; leaving out the time dependency for simplicity:

$$w_\mathrm{se1} = \frac{q_1^2}{2C_\mathrm{TJ1}} = \frac{C_\mathrm{TJ1}}{2(C_\mathrm{TJ1} + C_\mathrm{TJ2})^2} (C_\mathrm{TJ2}^2 v_\mathrm{s}^2 - 2C_\mathrm{TJ2} v_\mathrm{s} q_\mathrm{i} + q_\mathrm{i}^2) \tag{16}$$

$$w_\mathrm{se2} = \frac{q_2^2}{2C_\mathrm{TJ2}} = \frac{C_\mathrm{TJ2}}{2(C_\mathrm{TJ1} + C_\mathrm{TJ2})^2} (C_\mathrm{TJ1}^2 v_\mathrm{s}^2 + 2C_\mathrm{TJ1} v_\mathrm{s} q_\mathrm{i} + q_\mathrm{i}^2) \tag{17}$$

Adding these two equations we find for the total stored energy w_se:

$$w_\mathrm{se} = \frac{C_\mathrm{TJ1} C_\mathrm{TJ2}^2 + C_\mathrm{TJ1}^2 C_\mathrm{TJ2}}{2(C_\mathrm{TJ1} + C_\mathrm{TJ2})^2} v_\mathrm{s}^2 + \frac{1}{2(C_\mathrm{TJ1} + C_\mathrm{TJ2})} q_\mathrm{i}^2 \tag{18}$$

and for the change in stored energy during the tunnel event:

$$\Delta w_\mathrm{se} = \frac{1}{2(C_\mathrm{TJ1} + C_\mathrm{TJ2})} (q_\mathrm{i|after}^2 - q_\mathrm{i|before}^2) \tag{19}$$

If we now define δq_s as the charge that is transported through the source during the event then:

$$w_\mathrm{s} = \delta q_\mathrm{s} v_\mathrm{s} \tag{20}$$

and we can calculate the condition for tunneling:

$$w_\mathrm{s} - \Delta w_\mathrm{se} > 0. \tag{21}$$

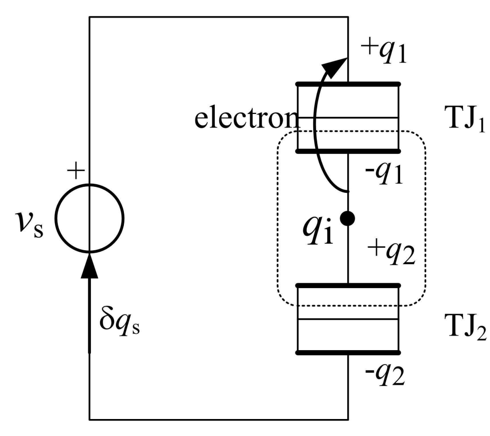

Fig. 6. An electron tunnels through junction 1 of the double-junction structure in the global view.

Now, the critical voltage for an electron to tunnel to or to tunnel from an initially uncharged island (the same case as we studied in the previous section) can be calculated, that is: $q_\mathrm{i|before} = 0$. First consider the tunneling through junction 1, see Figure 6. When the electron tunnels from the island, the island charge becomes e, that is: $q_\mathrm{i|after} = \mathrm{e}$. To find the value of the current through the source, δq_s, we consider the change in the charge on the negative side of junction 2:

$$\delta q_{\mathrm{s}} = (-q_{2|\mathrm{before}}) - (-q_{2|\mathrm{after}}) =$$

$$\frac{C_{\mathrm{TJ2}}}{C_{\mathrm{TJ1}} + C_{\mathrm{TJ2}}}(q_{\mathrm{i}|\mathrm{after}} - q_{\mathrm{i}|\mathrm{before}}) \tag{22}$$

If we, now, fill in the values of $q_{\mathrm{i}|\mathrm{befrore}}$ and $q_{\mathrm{i}|\mathrm{after}}$, we obtain for the tunnel condition (Equation 21) using Equations 19 and 20:

$$\frac{C_{\mathrm{TJ2}}}{C_{\mathrm{TJ1}} + C_{\mathrm{TJ2}}}\mathrm{e}v_{\mathrm{s}} - \frac{\mathrm{e}^2}{2(C_{\mathrm{TJ1}} + C_{\mathrm{TJ2}})} > 0 \tag{23}$$

And we obtain as a condition for tunneling

$$v_{\mathrm{s}} > \frac{\mathrm{e}}{2C_{\mathrm{TJ2}}}. \tag{24}$$

We can do the same calculation in the case that the electron tunnels towards the island through junction 2 and the result will be:

$$v_{\mathrm{s}} > \frac{\mathrm{e}}{2C_{\mathrm{TJ1}}}. \tag{25}$$

Again we find the same criterion as in case of the single-electron charging approach:

$$|v_{\mathrm{s}}| > v_{\mathrm{s}}^{\mathrm{cr}} \equiv \min\left(\frac{\mathrm{e}}{2C_{\mathrm{TJ1}}}; \frac{\mathrm{e}}{2C_{\mathrm{TJ2}}}\right) \tag{26}$$

The question we now seek to answer is whether this free-energy approach can be use to develop a circuit theory. Therefore we have to consider any circuit, so also the double junction circuit, as a system—and have to consider a reduction of energy in the circuit. We can say this, because in case of Coulomb blockade or Coulomb oscillations the free energy *is* expressed in energy delivered by circuit sources and the energy stored or retrieved from capacitors only. This poses a serious problem. Due to the combined validity of Kirchhoff's current law and Kirchhoff's voltage law, and formulated as a topological theorem called Tellegen's theorem, power and thus *energy is conserved* in any circuit. According to this the change in free energy as defined in Equation 7 must be zero always. In this case we will find a solution for the critical voltage (the solution of $\Delta F = 0$), but we cannot interpret $\Delta F < 0$ or $\Delta F > 0$ because *both* belong to invalid solutions of the circuit in the sense that at least one of the Kirchhoff laws is violated.

But there is more and it is again related to the absence of resistances in the circuits. The circuit solution for the double junction excited by the ideal voltage source, that is $\Delta F = 0$, involves unbounded currents. The currents in the circuit are represented by (Dirac) delta functions, and do not represent physical currents. In fact the problem that encounters in the free-energy approach description resembles the in circuit theory well known switched capacitors problem, in which a charged capacitor is connected by a switch to a second uncharged capacitor. Energy calculations shows that half of the initial energy is lost when the charge is divided over both capacitors after closing the switch. It looks as if the energy in the circuit is decreased. The standard solution to this classical network problem is that a resistor *must* be included to obtain a physical acceptable network (allowing only bounded currents). The resistor then absorbs the energy that seemed to be disappeared.

Again, we see that the energy concept becomes indistinct: the decrease in energy in the free-energy approach might just be there because we cannot include resistors. In addition to this, circuits including resistances, for example circuits including real sources, cannot be analyzed by this approach.

The dissipation of energy in the circuits inspired us to develop the impulse circuit approach that includes resistances from the very start.

III. Direct-Tunneling Based Approach

In this section the impulse circuit model will be discussed. It incorporates resistors in a general way and translates the quantization of charge into the quantization of current and of time during a tunnel event. This approach transforms global energy concepts into a concept using local voltages and allows necessarily also nonzero tunneling times (if resistors have to dissipate energy during a tunnel event the time to tunnel must be nonzero and current must be bounded). The approach is an extension of the direct-tunneling concepts that are used in the explanations of the working of the more classical nanoelectronic devices: the tunnel diode and the metal-insulator-metal tunnel device.

Both in case of the tunnel diode (Esaki diode) and the metal-insulator-metal (M-I-M) tunnel junction the main mechanism for tunneling is direct tunneling, also called direct band-to-band tunneling. Electrons from a filled band tunnel to empty states in an allowed band on the opposite side of the barrier without loosing energy, that is, conserving energy. Pictured in an energy-band diagram the tunneling electrons follow a horizontal path, see for example [23]. (Dissipative tunneling through traps in the barrier is a minor mechanism and will be neglected here.)

While during direct tunneling no energy is dissipated we *know* that both the tunnel diode and the M-

I-M tunnel junction do dissipate energy because there vi-characteristics ly in the first and third quadrant (for the power P holds: $P = iv \geq 0$). For these devices the explanation of the dissipation is found in dissipation mechanisms outside the tunnel barrier. For the M-I-M tunnel junction the mechanism is rather simple and known under the name *hot electrons*. After tunneling the electrons enter the metal at the positive side well above the Fermi-level, see Figure 7. Electrons appearing after tunneling above the Fermi level possess a high kinetic energy, and are called hot electrons [24]. If the tunneling electrons are at the Fermi level at the negative side then upon arriving at the positive side of the barrier they are "hotter" by just about ev_{TJ}. The hot electrons travel into the metal at the positive side thereby losing their excess energy. The two types of collisions responsible for this are electron-phonon and electron-electron collisions. The latter is important in the metal because of the high free-electron concentration. Repeated collisions drop the energy of all hot-electrons, and eventually, some distance away from the interface, they become part of the normal free-electron distribution. Because the possibility of an electron to be a hot electron only depends on the voltage across the tunneling junction v_{TJ} circuits with resistors can be included easily; with resistors included, at the tunneling junction the Fermi-levels and their difference change in time.

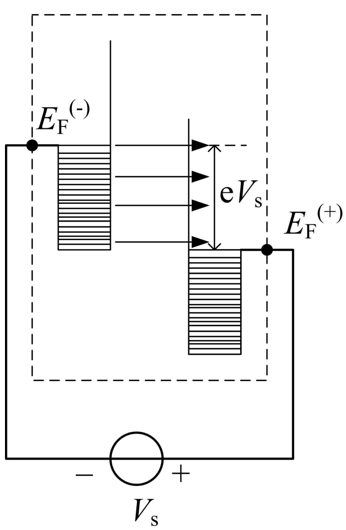

Fig. 7. Energy diagram for tunneling between two metals with fixed Fermi levels, that is, charging effects due to electron tunneling can be neglected. Tunneling is possible only when filled energy states are facing empty energy states. The electrical behavior is resistive; for small source voltages, the tunneling junction can be modeled as a linear resistor.

The impulse circuit model proposes the following formula for the critical voltage: if the voltage across the tunnel junction just before the tunnel event is $v_{\mathrm{TJ|before}}$, and the voltage across the tunnel junction just after the tunnel event is $v_{\mathrm{TJ|after}}$ then for the critical voltage across the tunnel junction holds:

$$v_{\mathrm{TJ}}^{\mathrm{cr}} = \{v_{\mathrm{TJ|before}} : v_{\mathrm{TJ|after}} = -v_{\mathrm{TJ|before}}\}; \tag{27}$$

in words: the critical voltage is the set of all the voltages across the tunnel junction before the tunnel event for which holds that the voltage across the tunnel junction *after* a tunnel event is the opposite of the voltage *before* the event. Using a transit time τ as the time the electron needs to pass from Fermi-level to Fermi-level, we can write:

$$v_{\mathrm{TJ}}^{\mathrm{cr}}(t,\tau) =$$

$$\{v_{\mathrm{TJ|before}}(t) : v_{\mathrm{TJ|after}}(t+\tau) = -v_{\mathrm{TJ|before}}(t)\}. \tag{28}$$

Realize that if the voltage across the junction equals the critical voltage than the transition time reduces to the tunneling time, that is, the time to pass the barrier. At the critical voltage the Fermi level before tunneling at one side is lying exactly opposite the Fermi level after tunneling at the other side.

Equation 27 is also known in the free-energy approach. It describes the critical voltage in the circuit if the tunnel time is zero, the current is unbounded, and the reduction of the free energy is zero; so there is energy conservation.

A physical interpretation of Equation 28 leads to an the extended version of the hot-electron model. Due to the description with resistors and currents the Fermi levels change dynamically. If a nonzero transit time is included the Fermi level changes during the tunnel event. During this event the position of the electron is unknown as is the voltage across the tunneling junction; the time is quantized, in the sense that, the voltage and charge before the tunnel event are known, the voltage and charge after the event are known, but the voltage and charge are not defined during the event. The interpretation of the tunnel condition is now that tunneling is possible if after tunneling the electron can still be a hot electron, see Figure 8. During the transition the (displacement) current is modeled with a impulsive delta function: $i_{\mathrm{TJ}} = \mathrm{e}\delta(t)$, showing the quantization of charge.

Now we look at the double junction structure, and consider a circuit consisting of an ideal voltage source v_{s} in series with a resistor R and two tunnel junctions having capacitances C_{TJ1} and C_{TJ2}. Because

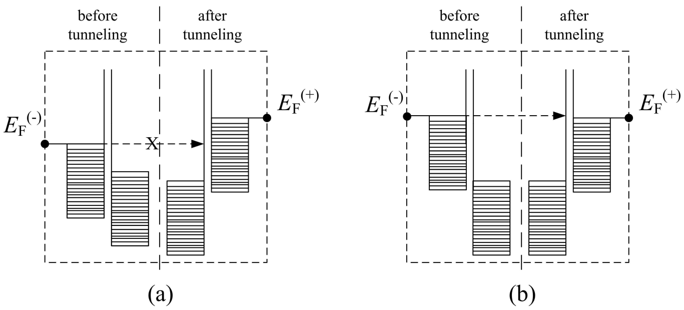

Fig. 8. Model for Coulomb blockade and tunneling in a tunnel junction driven by a current source having a very small value. (a): If the initial charge on the tunnel junction is $e/4$ the junction shows Coulomb blockade, because the electron (or electron wave) just after leaving the negative side and approaching the positive side experience moving Fermi levels in such a way that the Fermi levels at the negative side appear to be filled. (b): If the initial charge on the tunnel junction is just above $e/2$ the junction shows a tunnel event, the Fermi levels also change but the electron can still enter the positive side as a hot electron (it enters just above the Fermi level). The difference of the Fermi levels is a bit exaggerated for clarity.

the tunnel condition is now a condition involving only the local variables v_{TJ1} and v_{TJ2}, the condition is met if enough charge is brought to the capacitors. In the circuit the resistor defines the current as $i(t) = (v_{\mathrm{s}}/R)(\exp(-t/(RC))$ where v_{s} is the value of the constant voltage source, R the resistor value, and C the equivalent capacitor value in case $q_{\mathrm{i}} = 0$. Both junction capacitors will be charged with the same amount of charge at the same time. According to Equation 28, different tunneling times predict different critical values of the voltage and thus different critical values of the charge.

In short, the following possibilities are predicted; for a more elaborated exposition see [14]. First, we consider a zero tunneling time. If the steady-state voltage *across a tunnel junction* never exceeds $\mathrm{e}/(2C_{\mathrm{TJ}})$ the junction is in Coulomb blockade. If the steady state voltage across a tunnel junction exceeds $\mathrm{e}/(2C_{\mathrm{TJ}})$ the two junction structure will show Coulomb oscillations, with amplitude $\mathrm{e}/(2C_{\mathrm{TJ}})$. Second, when the tunneling time is nonzero the critical voltage across a junction will decrease, due to the amount of charge Δq transferred between the two capacitors *during the tunnel event* as a consequence of the current through the circuit and the tunneling time. The critical voltage will decrease until $\Delta q =$e, in which case the critical voltage of the junction is zero and the junction will tunnel continuously (no Coulomb blockade or Coulomb oscillations can be observed).

IV. Conclusion

In this paper, various approaches to modeling tunneling and Coulomb blockade using single-electron tunneling devices were reviewed and tested for their usefulness for designing nanoelectronic circuits that include these devices. The approaches differed in the way they incorporate the quantization of charge into a circuit theory. First, energy based theories were discussed, together often called orthodox theory of single electronics. In the approach using the single-electron charging energy the quantization of charge led to the quantization of the continuous spectrum of energy states associated with the capacitance of the small metal island. The transition from a device description to a circuit theory showed that resistors could not be incorporated, while the existence of resistors was necessary to come to a circuit theory. The approach using the reduction of the free-energy also lacked the resistors necessary to create a circuit theory. In the last section the impulse circuit model was discussed. It treated resistors in a general way and translated the quantization of charge into the quantization of current and of time during a tunnel event. Because only voltages across the tunnel junctions have to be calculated, both before tunneling and after tunneling, the use of circuit simulators, like SPICE, is possible. And makes such an approach very useful as a starting point for a nanoelectronic circuit theory.

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