Jitter Limitations on a Gigabit Copper Multi-Carrier System

Jan H. Rutger Schrader, Eric A.M. Klumperink, Jan L. Visschers[1], Bram Nauta
University of Twente, IC-Design Group,
P.O. Box 217, 7500 AE, Enschede, The Netherlands.
http://icd.el.utwente.nl
[1]National Institute for Nuclear Physics and High-Energy Physics,
P.O. Box 41882, 1009 DB, Amsterdam, The Netherlands.
E-mail: j.h.r.schrader@utwente.nl

Abstract—**The feasibility of a multi-carrier system for data-transmission over copper wires at gigabit rates is analyzed. More specific, we look at the effects of jitter and duty-cycle deviations on the SNR and error rate using Matlab simulations. Jitter causes crosstalk between the in-phase and quadrature channels of a given tone frequency. Given a certain rms jitter variance, low frequency tones can carry more bits than higher frequency tones for the same error rate. Duty-cycle deviations cause crosstalk both from other carrier frequencies and from quadrature channels. These problems seem so big that a PAM system is likely to be better choice for gigabit transmission.**

Keywords— **data-communication, jitter, multi-carrier modulation**

I. Introduction

Very high speeds have been achieved in the area of copper wire transmission. For example, using an equalized Pulse Amplitude Modulation (PAM) system, a bit rate of ~7Gbps has been reached over a distance of 10m at a Bit Error Rate (BER) of ~$1 \cdot 10^{-12}$ [1]. There is a continuing demand for higher bit rates. The bandwidth and signal-to-noise ratio of short cables can be high enough to allow this. This study explores the possibilities of adapting telephone modem and Digital Subscriber Line (DSL) techniques using Orthogonal Frequency Division Multiplexing (OFDM) to GHz bandwidths, possibly enabling >10Gbps transmission speeds over copper wires. In general, these systems are implemented using DSP techniques, but this is unfeasible for a bandwidth in the gigahertz range. Therefore the architecture has to be fundamentally different. We will introduce an architecture that solves some of these problems.

In this paper, Matlab simulations will be presented that give insight into the system-level trade-offs and possibilities of such a system. More specific, the effect of jitter and duty-cycle deviations on such a system will be analyzed. We will calculate SNRs and error rates for a number of different values for the rms jitter, and for a number of different duty-cycle deviations.

II. Arguments for Multi-Carrier

Transmission of signals over copper wires suffers from skin-effect and dielectric losses – resulting in a frequency-dependent attenuation and dispersion (=frequency-dependent propagation delay).

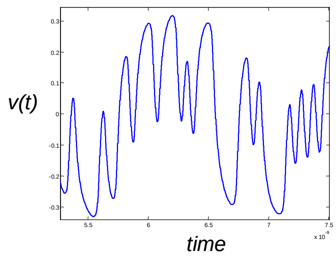

Figure 1 – Baseline wander

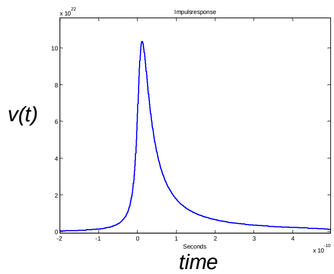

Figure 2 – Copper medium impulse response

If the signaling speed of a PAM signal is increased past the bandwidth of the channel, the (unequalized) received signal will show 'baseline wander' as shown in figure 1. In figure 2, the impulse response of a copper cable or PCB trace is shown. The very long tail causes intersymbol-interference (ISI). (More information on modeling these cables can be found in [4].) Lowering the PAM baud rate will decrease ISI, because the signal spectrum is moved to the lower frequencies where skin effect and dielectric losses have less effect. (This explains the effort being put into PAM4 signaling [1].) It is important to note that ISI is dependent of the baud rate and not the total bit rate. Multiplexing several bits into one symbol will decrease ISI while the total bit rate can remain the same. Each sub-channel has a much lower bit rate, making it possible to have much larger symbol duration. This is exactly what happens in multi-carrier modulation, so using such a system will decrease the demands on channel equalization. Furthermore, using multi-carrier techniques enables spectrally efficient modulation like QAM. Also a multi-carrier system is useful when the channel contains spectral nulls as is sometimes the case with PCB traces and connectors.

III. OFDM BASICS

The transmitted data is modulated on several orthogonal carriers. In order to avoid interference these have to comply with the orthogonality constraint, which is defined as

$$\frac{1}{T_{op}}\int_0^{Top} c_i(t)\cdot c_j(t)dt = \delta_{ij}, \quad \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases} \tag{1}$$

where $c_{i,j}(t)$ are the carriers and T_{op} is the length of the receiver integration period ('orthogonality period'). Candidates for tone frequencies f_c are harmonic frequencies n/T_{op}. Integration over exactly T_{op} delivers perfectly orthogonal carriers.

The total bit rate b_{tot} of such a system will be

$$b_{tot} = \frac{2}{T_s}\sum_{i=1}^{N_t} {}^2\log(N_{l,i}) \tag{2}$$

where $N_{l,i}$ is the number of DAC/ADC levels used on the i^{th} tone, N_t the number of tone frequencies in the system (where each tone is modulated with both an in-phase- and quadrature component) and T_s the total symbol length (optionally including guard time T_{gt} [2]).

IV. SYSTEM ARCHITECTURE

The multi-carrier communication system architecture that we will study is shown in figure 3. On the left, the transmitter is shown and on the right the receiver. We use (passive or active) mixers for multiplication of the data streams with the carrier signals. For the integration over the symbol period, an integrate-and-dump block is used which can be implemented using a capacitor and switches. ADCs and DACs can be added to provide more levels – and thus a higher spectral efficiency. One architectural advantage now becomes apparent: parallelization is used for the converters and integrate-and-dump blocks, which relaxes bandwidth requirements.

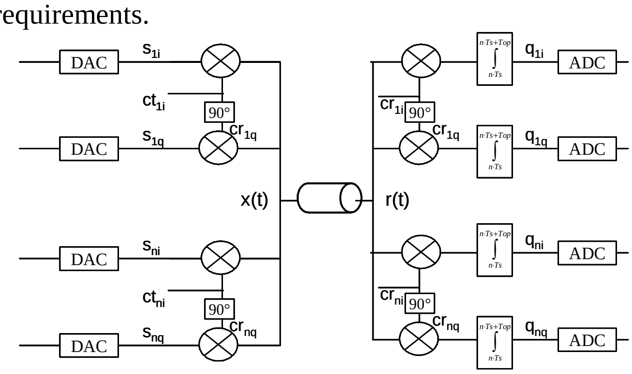

Figure 3 – System architecture

In-phase and quadrature carriers can be generated using a voltage-controlled ring-oscillator. The multiple phases coming from the ring oscillator can also be used to control the integrate-and-dump blocks. Of course a clock-and-data-recovery (CDR) block will also have to be included in the receiver.

This architecture puts a number of constraints onto the carrier waveforms that we can use. In an implementation with simple switching mixers, a sine wave on the local oscillator port will generate a square wave on the output. This will produce harmonics that fall onto other tone frequencies, creating unusable areas in the spectrum. A solution to the problem could be the use of harmonic rejection mixers [3]. Because of the above mentioned problems, it is likely that the best chances for such a system are for a low number of carriers.

The signals are defined as follows. The symbol streams $s_{ni}(t)$ and $s_{nq}(t)$, chosen from $\{-A_{max}, A_{max}\}$, are modulated onto the (in-phase and quadrature) carrier signals $ct_{ni}(t)$ and $ct_{nq}(t)$ and added up, resulting in the sum signal $x(t)$ which is put onto the transmission line. At the receiver side the received signal $r(t)$ is demodulated using a 'correlator receiver' consisting of a multiplication with locally generated (in-phase and quadrature) carrier signals $cr_{ni}(t)$ and $cr_{nq}(t)$ and an integrate-and-dump. The recovered symbol streams are $q_{ni}(t)$ and $q_{nq}(t)$.

The transmitted signal $x(t)$ and the received signal $r(t)$ can be described as follows:

$$x(t) = \sum_{n=1}^{Nt} ct_{ni}(t) \cdot s_{ni}(t) + ct_{nq}(t) \cdot s_{ni}(t), \tag{3}$$

$$r(t) = \int_{-\infty}^{\infty} x(\lambda) \cdot h(t-\lambda) d\lambda, \tag{4}$$

where $h(t)$ is the impulse response of the channel.

V. ANALYSIS

For the system described above we will analyze the impact of jitter and duty-cycle deviations. The multi-carrier receiver is a 'correlator receiver' (multiplication followed by integrate-and-dump) as opposed to the sampling receiver used in most PAM systems.

The analysis goal is to determine the variance in the integrator output as a function of variance in τ (time shift between transmitter and receiver), as illustrated in figure 4. Time offsets over the symbol, caused by jitter, lead to imperfect separation of the in-phase and quadrature component of the carrier. Duty cycle errors lead to other types of crosstalk.

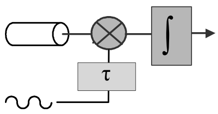

Figure 4 – Influence of τ

VI. SIMULATION RESULTS

A. *Impact of jitter*

We will estimate the influence of jitter on the error rate of the system. In order to analyze this it is recognized that this error rate is a function of the SNR. We will calculate an effective SNR as a function of jitter. It is assumed that the jitter coming from the PLL has a Gaussian time distribution with an rms variance of σ_t. Its size is determined by the PLL noise and loop bandwidth. For a state-of-the-art LC-based PLL this is currently in the order of 1ps rms. For a ring oscillator it will be around 5ps rms. The effective SNR as a function of this jitter is expected to be dependent on carrier frequency f_c and number of levels N_l used in modulation. We will do the following simulations:

- f_c=0.5GHz, N_l=4, σ_t=10ps
- f_c=2.5GHz, N_l=4, σ_t=10ps
- f_c=0.5GHz, N_l=16, σ_t=10ps

A jitter of 10ps is taken because this will enable us to detect errors in a relatively short simulation (time step size=1ps). Now let's look at the results of the statistical Matlab simulations (500 symbols) and focus on the error rate for the in-phase component. We use only a single carrier frequency for the sake of simplicity, and analyze crosstalk between in-phase and quadrature component as a function of variance in τ caused by jitter. The jitter has been modeled by adding a Gaussian random variable to the receiver time axis. Furthermore, the channel response is set to ideal (impulse response is a Dirac pulse). The SNR per symbol values have been calculated by sampling the data, demodulating the data and applying a normal distribution fitting routine. This routine estimates the amplitude variance σ_A of the integrator output which is used to obtain the SNR per symbol SNR_{sm} as

$$SNR_{sm} = \left(\frac{h}{2\sigma_A} \right)^2 \tag{5}$$

where h is the distance between two adjacent levels. Next, the probability of error P_e for the in-phase component can be calculated as

$$P_e = Q\left(\sqrt{SNR_{sm}}\right). \tag{6}$$

We will use histograms of the integrator output to graphically show the variance σ_A.

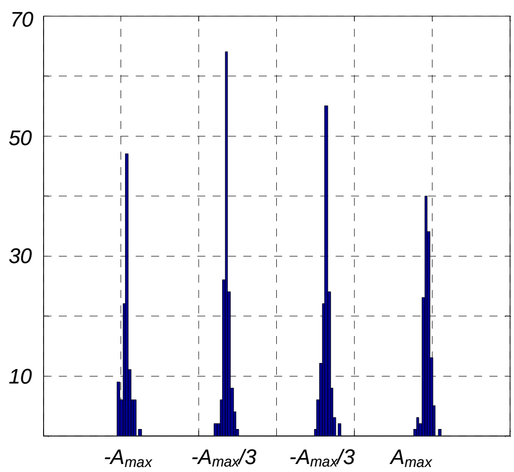

Figure 5 – Output hist. (jitter σ_t=10ps, f_c=0.5GHz, N_l=4)

When we add a jitter of σ_t=10ps in the receiver generated carrier, and the frequency and number of levels are set at f_c=0.5GHz and N_l=4 levels (=2 bits) we can transmit data relatively well. This is shown in figure 5. From the simulation, the calculated SNR_{sm}=25dB, which gives an excellent Pe«$1\cdot10^{-12}$.

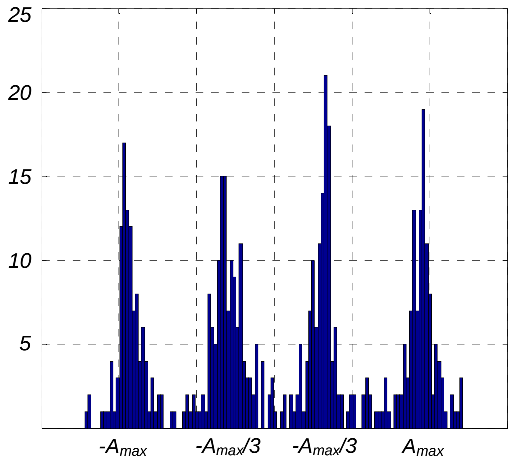

Figure 6 – Output hist. (jitter σ_t=10ps, f_c=2.5GHz, N_l=4)

However when we try to increase the frequency to f_c=2.5GHz (number of levels remains N_l=4), the jitter starts to have a severe impact, causing SNR_{sm}=10dB, which gives a bad Pe=$1\cdot10^{-3}$. (This is shown in figure 6.)

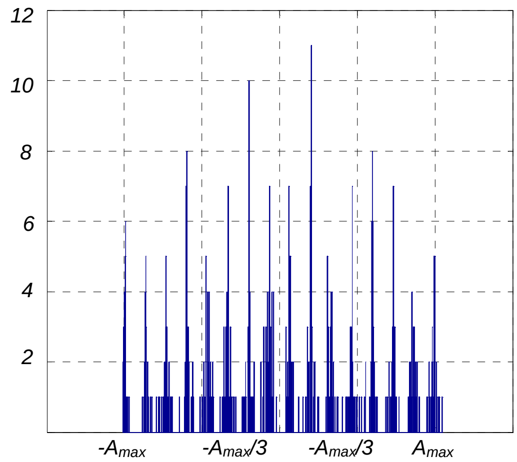

Figure 7 – Output hist. (jitter σ_t=10ps, f_c=0.5GHz, N_l=16)

A comparable jitter impact can be seen (in figure 7) when we do not increase f_c (remains at f_c=0.5GHz) but rather increase the modulation depth to N_l=16 levels. The signal to noise ratio per symbol SNR_{sm}=12dB, which gives an equally bad Pe=$1\cdot10^{-6}$.

The observation to be made is that, for a given jitter rms variance, low frequency tones can carry more bits than higher frequency tones for the same error rate.

B. *Impact of duty-cycle*

Another circuit non-ideality that is a threat to the carrier orthogonality is duty cycle distortion. The correlator output is dependent on the duty cycle of the signal, so deterministic deviations and/or variations in the duty cycle can disturb the orthogonality. We simulate these effects for a system with two tone frequencies (f_2=2f_1) using squarewave carriers, where each frequency is modulated with both in-phase and quadrature components. (Therefore the total number of carriers is four.) The duty-cycle *d* of the receiver generated in-phase carrier $cr_{1,i}$ at f_1 is varied, while the duty-cycle of all the transmitted carriers is exactly 50%. Below, a short summary of the simulation setup is given.

Transmitter carriers:

- $ct_{1,i}$: f_c=f_1, in-phase component
- $ct_{1,q}$: f_c=f_1, quadrature component
- $ct_{2,i}$: f_c=f_2, in-phase component
- $ct_{2,q}$: f_c=f_2, quadrature component

Receiver carrier:

- $cr_{1,i}$: f_c=f_1, in-phase component, deviations in duty-cycle

We will calculate the following correlations as a function of the duty-cycle of $cr_{1,i}$:

- $corr_1$ = between $cr_{1,i}$ and $ct_{1,i}$
- $corr_1$ = between $cr_{1,i}$ and $ct_{1,q}$
- $corr_1$ = between $cr_{1,i}$ and $ct_{2,i}$
- $corr_1$ = between $cr_{1,i}$ and $ct_{2,q}$

These are all possible correlation for this two-tone system.

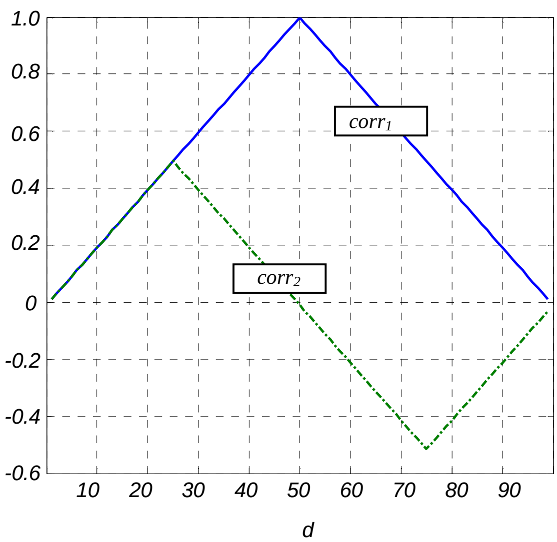

Figure 8 – $corr_1$ and $corr_2$ versus duty cycle *d*

Figure 8 shows the correlation of $cr_{1,i}$ with the wanted transmitted signal $ct_{1,i}$ and with the quadrature component $ct_{1,q}$ at that same frequency (respectively $corr_1$ and $corr_2$). Figure 9 shows the correlation of $cr_{1,i}$ with the double frequency carriers $ct_{2,i}$ and $ct_{2,q}$ (respectively $corr_3$ and $corr_4$). The values have been normalized to one.

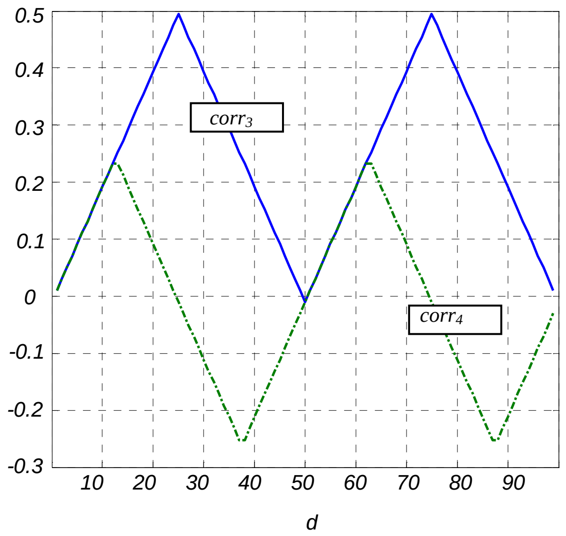

Figure 9 – $corr_3$ and $corr_4$ versus duty cycle d

All deviations from 50% duty-cycle introduce inter-carrier interference (crosstalk) and lead to a limited SNR, as shown in figure 10. The peak at 50% corresponds to an infinite SNR.

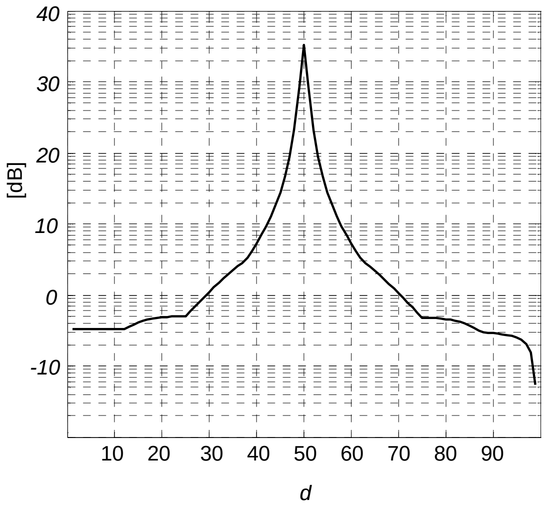

Figure 10 - SNR as a function of duty cycle d

The error rate as a function of the duty cycle is shown in figure 11. The observation to be made is that a multi-carrier system is very sensitive to duty-cycle variations, with the error rate becoming unacceptably high for >5% variation.

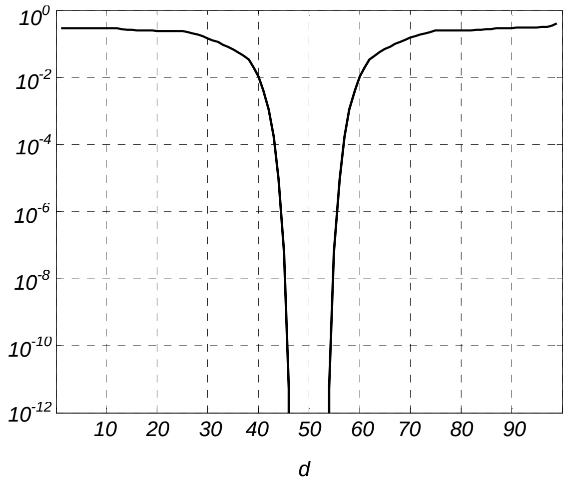

Figure 11 – error rate P_e as a function of duty cycle d

VII. Conclusions

A transceiver architecture for gigabit multi-carrier transmission over short copper wires has been presented. It offers advantages concerning DAC and ADC requirements by using parallelization. A short feasibility analysis has been made. Harmonics caused by switching mixers hinder the use of a large number of carriers. A state-of-the-art PLL rms jitter will impose a serious limit on the bit rate of a gigabit multi-carrier system. For a given rms jitter variance, low frequency tones can carry more bits than higher frequency tones for the same error rate. Furthermore, a duty cycle deviation of more than 5% causes the error rate of a two frequency multi-carrier system to drop below acceptable numbers. It is demonstrated that a lot of problems can be expected from the above mentioned sensitivity to jitter and duty cycle errors. It seems that a PAM system is the better choice for very high (giga-) bit rates on copper media.

References

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