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A qualitative analysis of the distribution of bed-surface elevation and the characteristics of associated deposits for subaqueous dunes

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ABSTRACT

This paper analyses controls on the probability distribution of bed-surface elevations, P_s, and the structure and texture of the associated deposits, in dune-forming conditions. It is important to understand these controls in order to develop predictors required to implement a new theory that is based on a probabilistic approach of the Exner equation for sediment continuity, and to improve the interpretation of sedimentological records of fluvial origin. Experiments were conducted in three flumes of different sizes, with flow depth ranging from 0.15 to 0.87 m, flow velocity ranging from 0.5 to 0.84 m s^{-1}, and with sand- to gravel-dominated mixtures. Distributions of bed-surface elevations were measured from time-series and/or successive bed profiles. Vertical profiles of bed composition (i.e. vertical sorting) and/or structure (i.e. cross-sets) of the deposits were analysed.

Results show that the dimensionless bed shear stress and vertical sorting are the major controls on the range and shape of the non-dimensional P_s curve. Here, the dimensionless bed shear stress is used as a component of the sediment transport stage. Low values of sediment transport stage produce P_s curves that are short-ranged and quite evenly spread around the average bed level. The presence of a coarse bed layer, which affects the composition of the transported sediment, has a similar effect on the P_s curve. Otherwise, the P_s curves tend to be skewed toward the lowest values of bed surface elevation. Interactions between sediment transport stage, vertical sorting, dune height, and the P_s curves, mean that the last of them can be partially reconstructed from the analysis of the geometry and texture of cross-sets. The distribution of elevation of cross-set lower boundaries mimics the P_s curve toward its lower limit. The ratio of mean dune height over flow depth indicates approximately the upper limit of the P_s curve, and these variables can be estimated via cross-set thickness distribution. It is hoped that this study will help develop a comprehensive theory that could predict sediment transport from the characteristics of dunes and their deposits, and vice-versa.

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INTRODUCTION

Dunes are common bedforms in rivers, and changes in their geometry as they migrate under various flow conditions affect the bed morphology and the characteristics of their preserved deposits (e.g. Allen, 1982). The nature of dunes is of interest to scientists working with different perspectives, e.g. engineers concerned about river-bed erosion or sedimentologists eager to interpret the fluvial record. For that reason, the authors have independently developed similar data bases on dune characteristics, although pursuing different objectives, such as dune migration and formation of cross-sets (Leclair *et al.*, 1997; Leclair, 2000, 2002; Leclair & Bridge, 2001) or vertical sorting to improve sediment continuity concepts for non-uniform sediment (Blom, 2000; Blom *et al.*, 2001, 2003). The present study stems from a shared curiosity about dune-migration processes and their effects on bed topography and the characteristics of associated deposits.

The two studies discussed here were conducted separately, so many differences as well as similarities exist between them. Differences occurred in the:

1 choice of flow and sediment conditions for the experiments, resulting in large ranges in flow depth, velocity and sediment transport stage;

2 sampling scheme for dune deposits, which favoured structural (Leclair) or textural (Blom) analysis;

3 overall analytical procedures, which highlighted different aspects of the relationships between sediment transport and the characteristics of subaqueous dunes and their deposits (more below).

The major similarities arise from a common particular attention to the occurrence of the deepest dune troughs, as these represent the possible lower boundaries of cross-sets, as well as the location of the coarser particles. Hence, extensive, high-resolution sampling of variation in bed-surface elevation was produced, as well as detailed measurements of dune geometry, including matching variables rarely available in other data sets, such as the height of individual dunes and the depth of their associated trough scour relative to mean bed level. Moreover, the differences between experimental settings provide an interesting range of conditions that might have never been planned otherwise. This paper integrates these two studies by applying some of the analytical methods developed with one data set to the other set, and presents new insights resulting from this integration effort.

Cross-set formation model

Figure 1 illustrates the concept of cross-set formation at a given point. The variability of the height and trough-scour depth of successive bedforms can produce a stack of cross-sets, even without net deposition (Paola & Borgman, 1991; Leclair, 1997, 2002). In the Paola-Borgman model (1991) for the probability density function (PDF) of cross-set thickness, s,

$$p(s) = (ae^{-as}(e^{-as} + as - 1)/(1 - e^{-as})^2 \qquad (1)$$

the parameter a represents the mean and standard deviation of trough-scour depth, ts, because by definition, $a = 1/\beta$ and $\beta = ts_{sd}^2/ts_m$. Leclair & Bridge (2001) proposed a modified Paola-Borgman model

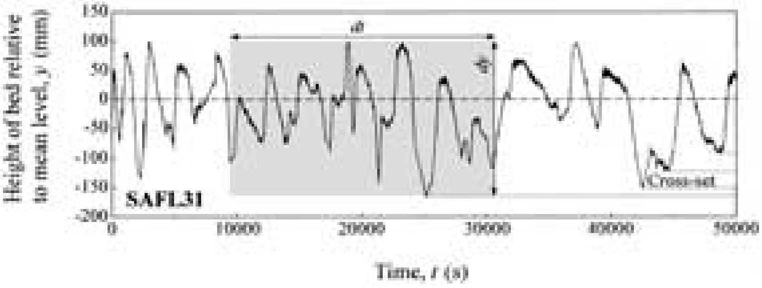

Fig. 1 Time-series of variation in bed-surface elevation from an experiment of Leclair (2000). Shaded block illustrates the concept behind the probabilistic Exner sediment continuity equation of Parker *et al.* (2000), based on the probabilistic nature of variations of bed-surface elevations owing to the migration of bedforms. These variations could be either a function of time or distance, as long as the flow conditions are steady and the bed-elevation series is stationary.

where the predicted mean cross-set thickness, s_m^{pred}, is defined by

$$s_m^{pred} = lr/c + 1.64493/(h_{sd}/ts_{sd})a \tag{2}$$

where l is mean dune length, r is mean aggradation rate, c is mean dune migration rate, h_{sd} and ts_{sd} are the standard deviation of dune height and trough-scour depth below mean bed level, respectively, and $a = 1/\beta$, estimated here from the gamma function describing dune-height probability density distribution. Therefore, the distribution of observed cross-set thicknesses reflects the distributions of dune height and trough-scour depth, and both of these contribute to the variation in bed topography that will be considered in the following section.

Sediment continuity models

Fundamental to predicting morphological changes is the Exner equation of sediment continuity, which can be expressed in the form

$$(1 - \lambda)(\partial\eta/\partial t) = D - E \tag{3}$$

where λ is the bed porosity, η is the average bed level, t is time, D is the volume of sediment deposited on to the bed, per unit area and time, and E is the volume of sediment entrained from the bed (Parker *et al.*, 2000, equation 11). Parker *et al.* (2000) derived a probabilistic form of the Exner equation for sediment continuity, based on the probabilistic nature of variations of bed-surface elevations due to the migration of bedforms (Fig. 1). For uniform sediment, they derived

$$(1 - \lambda)(\partial P_s/\partial t) = D_e - E_e \tag{4}$$

where P_s is the probability distribution of bed-surface elevations (e.g. Parker *et al.*, 2000, Fig. 7), and D_e and E_e are elevation-specific values of deposition and entrainment densities, respectively, defined such that $D_e\mathrm{d}x\mathrm{d}z$ and $E_e\mathrm{d}x\mathrm{d}z$ are the volumes of sediment deposited and entrained from a bed element with sides dx and dz (Parker *et al.*, 2000, equation 21). This new conceptual framework brought up the need for: (i) formulations for the elevation-specific deposition and entrainment densities for uniform and non-uniform sediment; and (ii) an understanding of the controls on the shape of the P_s curve in the case of dunes.

Recent developments

A first step towards new formulations for the elevation-specific deposition and entrainment densities in dune conditions and, as such, toward the development of a new sediment continuity concept for non-uniform sediment was accomplished by Blom *et al.* (2001). They considered the evolution of the sorting profile of tracer particles in uniform sediment. Then, they considered the possible controls on the shape of the P_s curve when the bed is covered with dunes. The active bed was defined as the range of bed-surface elevations that are exposed to the flow, and this definition is used in the present paper. Their experiments showed that for similar discharge, the P_s curve covered different ranges of bed-surface elevations, depending on the initial vertical sorting (more below). They also related the P_s curve to the vertical sorting in dune deposits and their results indicate that the coarser grains were usually associated with the dune-trough levels or the lower elevations of the active bed, that is, the initial vertical sorting has been modified by the dune-migration processes.

In the experiments of Blom *et al.* (2001), it seems that the difference in total volume of each size of sediment in the active bed affected the shape of the probability distribution of bed heights for a given discharge: the range of values for bed-surface elevation was larger in run B2, where an initial coarse bed laid over sediment composed only of the finest fraction, than in run A2, where a coarse bed laid over the tri-modal mixture (see details on grain-size distribution in Method section). Such results stress the importance of investigating the control of characteristics of the sediment mixture on the shape of the P_s curve for similar flow conditions. In addition, the P_s curve typically increased in its range of bed-surface elevations with increasing discharge (Blom *et al.*, 2003). The predictive possibilities of equation (4) require a better understanding of how the P_s curve changes with temporal variation in flow conditions (e.g. flow depth, velocity, energy slope; see Parker *et al.*, 2000, equation 27).

The goal of this paper is to take advantage of the existence of two new, parallel data sets on dune migration (Blom *et al.*, 2003; Leclair, 2000) to:

1 investigate the controls on the P_s curve by comparing results from experiments conducted under different flow and sediment conditions;
2 identify which parameters of the P_s curve could be estimated from the geometry and grain sorting of cross-sets;
3 attempt to set bases for the development of a comprehensive theory of dune bed-surface elevation PDF linked to cross-set thickness PDF.

METHODS

Experimental methods

The authors independently conducted experiments under dune conditions in sediment-recirculating flumes:
1 BU runs at Binghamton University, NY (length 7.6 m, width 0.6 m);
2 SAFL runs at Saint-Anthony Falls Laboratory, University of Minnesota (length 76 m, width 2.7 m);
3 Runs A1, A2, B1, B2, and T10 at WL/Delft Hydraulics (WL/DH), in The Netherlands (length 50 m, width 1 or 1.5 m).

Extensive descriptions of the experimental designs are given by Blom & Kleinhans (1999), Blom (2000), Leclair (2000, 2002) and Blom *et al.* (2003). The present paper considers a selected set of experiments from these studies. Flow conditions of the selected experiments are summarized in Table 1. A trimodal mixture of sediment was used for the A and B experiments at WL/DH, a sediment mixture from the River Rhine was used for run T10, and moderately well-sorted and poorly sorted sediment were used for the BU and SAFL experiments, respectively (Fig. 2).

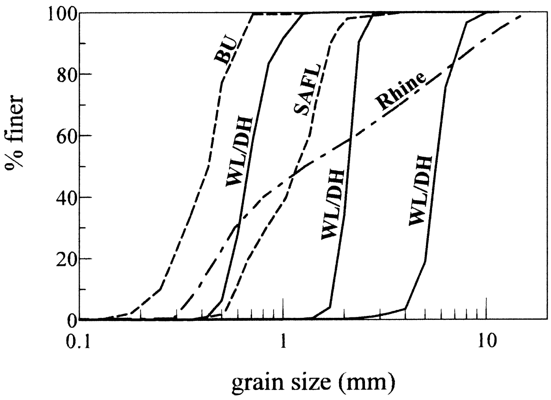

Fig. 2 Grain-size distribution in the experiments at Binghamton University (BU), Saint-Anthony Falls Laboratory (SAFL), and in runs T10 (Rhine) and A1-A2-B1-B2 at WL Delft Hydraulics (WL/DH).

The BU bed-elevation data, as well as all SAFL data, come from at-a-point time-series (Fig. 1; Leclair, 2000, 2002). The BU data on dune height and trough-scour depth below mean bed level were determined from successive bed-elevation profiles (Leclair, 2000, 2002), as were all WL/DH data (Blom, 2000; Plate 1). In Table 1, the symbols h_m and ts_m are the mean values, for a given run, of h_i and ts_i, which are the height and trough-scour depth of an individual dune at a given location on the bed profile and/or at a given time. Similarly, the ratio ts_m/h_m is the mean value of ts_i/h_i that indicates at which bed level an individual dune is migrating (e.g. Leclair, 2002). The ratio ts_m/h_m is equivalent to the mean dune-shape factor (compare values in Table 1 for WL/DH runs with data in Blom *et al.*, 2003, Table 2).

Cross-set thicknesses on sediment peels were measured for BU and SAFL experiments and mean cross-set thickness, s_m^{obs}, was computed from these measurements (Table 1; Leclair, 2001, 2002), whereas vertical profiles of the bed composition were described for WL/DH runs (Blom *et al.*, 2003). No sediment peels were made at WL/DH and it was not possible to perform a reliable quantitative description (e.g. using image-analysis software) of the vertical sorting in deposits from BU and SAFL peels. Finally, sediment transport rate was not measured at BU and SAFL.

Computations

The Froude number was computed as

$$Fr = U/(gd)^{1/2} \tag{5}$$

where U is mean flow velocity, g is acceleration due to gravity and d is mean flow depth. The spatially averaged bed shear stress was computed as

$$\tau_o = \rho g d S \tag{6}$$

where ρ is fluid density, and dimensionless spatially-averaged bed shear stress was computed as

Table 1 Experimental conditions and results for selected runs from Blom *et al.* (2003) and Leclair (2000).

Run	d (m)	U (m s^{-1})	D_{50} (mm)	Fr	S (10^{-3})	Ω	h_m/d (mm)	h_m (mm)	h_{sd} (mm)	h_{sd}/h_m	ts_m (mm)	ts_{sd}	ts_m/h_m ($x-1$)	h_{sd}/ts_{sd}	a (mm^{-1})	s_m^{pred} (mm)	s_m^{obs} (mm)
A1	0.154	0.64	TriM*	0.52	2.0	0.33	0.11	17	5.4	0.30	−8	2.6	0.47	2.0	0.58	1.4	ND
A2	0.320	0.83	TriM	0.47	1.8	0.60	0.15	49	15.7	0.32	−29	14.2	0.59	1.1	0.198	7.5	ND
B1	0.155	0.63	TriM	0.51	1.9	0.29	0.12	18	4.7	0.26	−9	2.4	0.50	1.95	0.81	1.1	ND
B2	0.389	0.69	TriM	0.35	2.2	0.89	0.31	122	32.9	0.26	−73	29.2	0.59	1.13	0.11	13.2	ND
BU9	0.15	0.5	0.43	0.40	3.7	0.94	0.29	43.2	16.9	0.39	−21.7	21.6	0.50	0.78	0.13	16.2	17.9
BU14	0.15	0.6	0.43	0.50	4.5	0.95	0.37	54.9	24.6	0.44	−38.3	24.6	0.69	1.0	0.077	21.4	15.1
BU21	0.15	0.75	0.43	0.60	4.1	0.95	0.32	48.2	21.0	0.43	−31.0	23.2	0.64	0.91	0.10	18.1	23.4
SAFL27	0.19	0.84	0.81	0.60	2.6	0.89	0.35	67	34	0.50	−50	37	0.74	0.9	0.07	26.1	26.1
SAFL29	0.21	0.5	0.81	0.34	3.0	0.90	0.36	76	43	0.56	−35	46	0.46	0.95	0.064	27.1	27.1
SAFL31	0.53	0.6	0.81	0.26	2.0	0.94	0.26	137	53	0.38	−76	75	0.55	0.71	0.13	17.8	32
SAFL32	0.54	0.8	0.81	0.35	2.0	0.94	0.21	115	45	0.39	−62	44	0.54	1.03	0.06	26.6	20
SAFL33	0.87	0.8	0.81	0.27	2.1	0.97	0.15	128	48	0.37	−75	58	0.59	0.83	0.04	49.5	32
T10	0.193	0.59	Rhine*	0.43	1.2	0.38	0.09	17	ND	ND	ND7	ND	0.41	ND	ND	ND	ND

*See Fig. 2; TriM = Trimodal mixture.

$$\theta = \tau_o/(\sigma - \rho)gD_{50} \qquad (7)$$

where σ is sediment density and D_{50} is median sediment grain size. Then, with θ_c being the value of θ at the threshold of sediment motion, a sediment-transport stage, Ω, was calculated as

$$\Omega = 1 - (\theta_c/\theta) \qquad (8)$$

This probably appears to be an unusual form of excess shear stress, but it is intended to be to the format used in Gill (1971), for analysis purposes. Predicted mean cross-set thickness, s_m^{pred}, was computed from equation (2). Data on experimental conditions and geometrical characteristics of dunes and cross-sets are in Table 1.

RESULTS

Bed-elevation probability distribution and dune geometry

Effect of initial vertical sorting

The present analysis provides a new illustration of the results from Blom *et al.* (2003) where different P_s curves were produced from runs with similar discharge but different initial vertical sorting (see e.g. runs A2 and B2 in Fig. 3). The wide range of bed-surface elevations in run B2 was due to mean dune height and trough-scour depth being more than twice as large as in run A2 (Table 1; Fig. 4). Figure 4 shows that the distribution of ts_i/h_i is more even relative to mean bed level (i.e. where $ts_i/h_i = -0.5$) in run A2 than in run B2. Moreover, Fig. 4 shows that many individual dunes with $h_i/d < 0.3$ in run B2 (i.e. smaller than the largest dunes of run A2) have trough scours attaining lower bed-surface elevations than they did in run A2 (compare dots for $ts_i/h_I < -0.5$ between the two graphs in Fig. 4). The reason for this variability in dune height and trough-scour depth between runs is because all size fractions in run B2 are being fully transported, whereas in run A2, partial transport prevailed and a coarse bed layer was present below the dunes. This new analysis highlights the interdependency between vertical sorting, grain-size-selective sediment transport, dune height distribution and P_s curve.

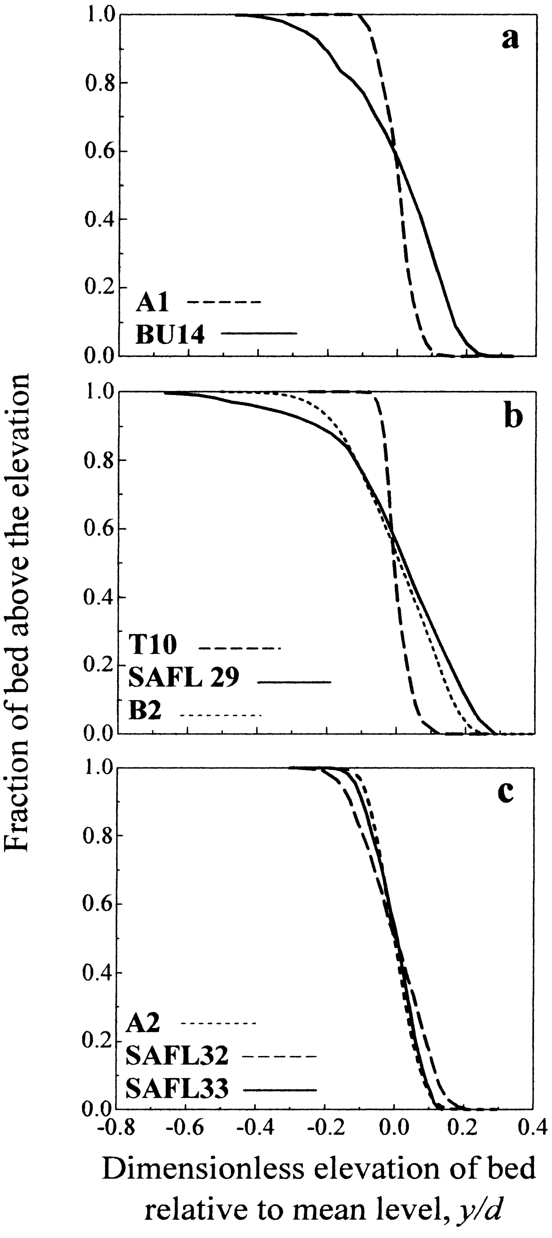

Fig. 3 Function P_s (y/d) denoting fraction of bed record at a point above bed surface elevation, y; where y is measured relative to mean bed level, which = 0, and non-dimensionalized using mean flow depth, d.

Effect of sediment mixture

Even without initial sorting, matching runs with different sediment mixtures but quite similar flow depths and velocities also produces different

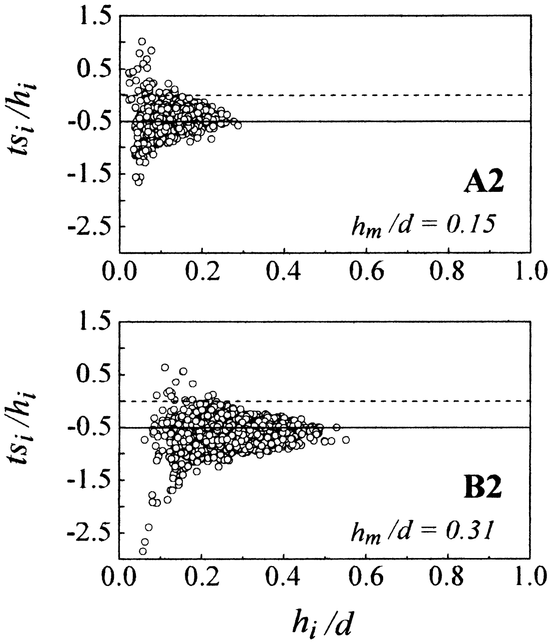

Fig. 4 Non-dimensionalized height of individual dunes, h_i/d versus ratio of trough-scour depth/dune height, ts_i/h_i, which indicates the relative elevation of the dune for experiments A2 and B2 with similar discharge but different initial vertical sorting (see text for details, and Blom *et al.*, 2003). The solid line at −0.5 indicates that dune-scour depth is half the dune height. Dunes migrating completely above mean bed-level plot above the dashed line.

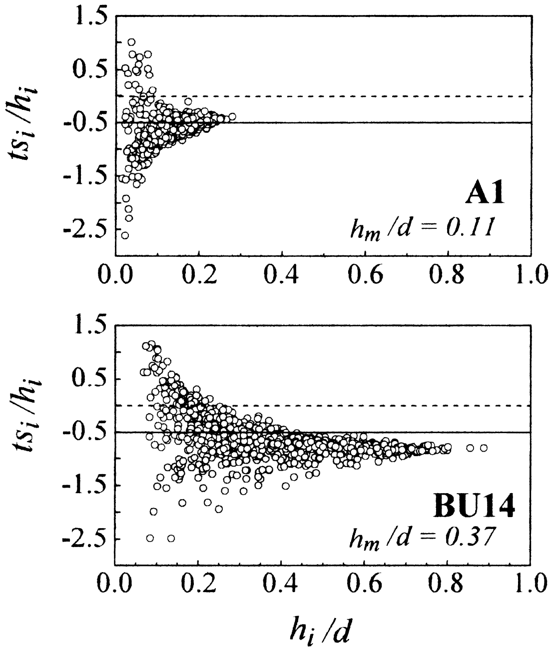

Fig. 5 Non-dimensionalized height of individual dunes, h_i/d, versus ratio of trough-scour depth/dune height, ts_i/h_i, which indicates the relative elevation of the dune for experiments A1 and BU14 with similar flow depth and velocity, but different sediment grain size. The solid line at −0.5 indicates that dune-scour depth is half the dune height. Dunes migrating completely above mean bed-level plot above the dashed line.

probability distributions of bed-surface elevation, P_s (runs A1 and BU14, Fig. 3a; runs T10 and SAFL29, Fig. 3b; Table 1). The range of values of dimensionless bed-surface elevation is smaller in the case of run A1 and run T10, which are experiments with coarser sediment mixtures, smaller values of energy slope, and hence lower sediment transport stages, than their respectively associated runs B14 and SAFL 29. Furthermore, the shape of the P_s curves for runs BU14 and SAFL29 is not spread evenly around mean bed-surface elevation, but is more skewed toward the low elevations than for their matching runs with a coarser sediment mixture. This is because significant differences in dune height and trough-scour depth are associated with the change in sediment transport stage: e.g. in run A1 (Fig. 5), the largest dunes of the distribution plot about evenly above and below the –0.5 line, whereas in run BU14 (Fig. 5) the large dunes ($h_i/d > \approx 0.4$) all have a ratio of trough-scour depth over dune height, $ts_i/h_I < -0.5$. The fact that dunes may or may not erode the bed at their troughs toward deeper levels depends on the vertical sorting profile and leads to different distributions of elevations relative to the mean bed-surface elevation, thus affecting the shape of the P_s curve.

Effect of mean dune height/flow depth

In the present experiments, the ratio of mean dune height over flow depth (h_m/d) ranges from 0.09 to 0.37 (Table 1) and these values are comparable to those observed in other flumes and rivers (e.g. Bridge & Jarvis, 1982; Iseya, 1984; Gabel, 1993; Mohrig & Smith, 1996). The interdependency between sediment transport stage, the P_s curve and dune height is illustrated by the variation of the ratio of mean dune height over flow depth (Fig. 3 and Table 1). Runs with similar sediment transport stage produce similar P_s curves: the curves of runs A1 and T10 reflect very well a low sediment transport stage, and runs with sediment transport stages > 0.90 (all BU runs and SAFL29) show only moderate variation in the range and shape of their P_s curves. Even runs such as B2 and SAFL29, which owe their similar values of sediment transport stage to different combinations of d, S and D_{50} (see equations 6 and 7), produce very similar P_s curves. The P_s curves for the experiments with the largest flow depths, however, do not reflect their high sediment transport stage. Indeed, the P_s curve for run SAFL33 is similar to those of runs A2, A1 and T10, despite its sediment transport stage of 0.97. Runs SAFL 32 and 33 have a relatively low ratio of mean dune height over flow depth, slightly above those for runs A1 and T10. Otherwise, it is worthy of note that the difference between the P_s curves of runs B2 and SAFL29 consists essentially in the extent of the curve toward the low-elevation values, which seems to be related to the difference in the variability of dune height, with $h_{sd}/h_m = 0.26$ and 0.56, respectively. No strong relationship (i.e. $r^2 = 0.49$) between h_{sd}/h_m and h_m/d was found in our combined set of data, however, suggesting that the real control might be the vertical sorting.

Dune deposits as clues for reconstructing bed-elevation probability distribution

Potential for preservation of cross-sets

For the WL/DH experiments, the model predicts a mean cross-set thickness that is about equal to, or much smaller than, the largest grains in the sediment mixture (i.e. *c.* 10 mm). Therefore, no or very few real cross-sets could be preserved in these runs. This limitation on the preservation of cross-sets is due to the low variability of dune height, i.e. to the value of the coefficient of variation of dune height (h_{sd}/h_m), which averages 0.28 for the WL/DH experiments, compared with 0.43 at BU and SAFL (Table 1). This situation for the WL/DH experiments is likely to be due both to the low conditions of sediment transport stage and to the formation of a coarse bed layer at the base of dunes. A similar case occurred at the North Fork Toutle River, however, where gravel dunes with $h_{sd}/h_m \approx 0.23-0.3$ left no cross-set, under a sediment transport stage of about 0.62–0.79 (estimated from Dinehart, 1992). In another published case, deposits from the Calamus River (Leclair & Bridge, 2001) were formed by dunes with $h_{sd}/h_m \approx 0.40-0.53$ and sediment transport stage of about 0.90–0.94 (estimated from Gabel, 1993). These values are in the same range as those from BU and SAFL experiments with preserved cross-sets. The present results therefore would seem to indicate that most preserved dune (and only dune) deposits record events with high sediment transport stages.

Cross-sets and vertical sorting

The interface between the dune deposits and the underlying sediment may indicate the nature of bedload transport that prevailed during their migration (assuming no change in sediment mixture occurred in the system). In the WL/DH experiments, the bedload material was always finer than the original material because conditions of selective transport prevailed (Blom *et al.*, 2003). This was clearly not the case in the BU and SAFL experiments. On sediment peels of deposits left by dunes in BU runs (moderately well-sorted sediment; Fig. 6), the cross-sets have more relief, indicating that they are composed of coarser grained sediment than the underlying non-transported sand (i.e. the dune deposit has larger pores in which the epoxy-resin flows). Note that the base of the deposit remains at about the same depth, although it is defined by different dunes (Fig. 6). This is because bed-height variation in time (e.g. Fig. 1), and hence the elevation of the deepest scours, are very similar at any given point along the profile (Leclair, 2000). Plate 2 shows that the dune deposit formed during run SAFL33 (poorly sorted sediment) is, like that of the BU runs (Fig. 6), coarser than the underlying sediment. As in the WL/DH experiments, however, coarser grains are found mostly at the base of the deposit (Plate 2). This is because coarse grains tend to settle at the base of individual dunes (Blom *et al.*, in press; Kleinhans, 2001), and because the very deepest dunes in a series are usually partially preserved (Leclair, 2000). Otherwise, in the upper part of the deposit, the vertical sorting varies markedly

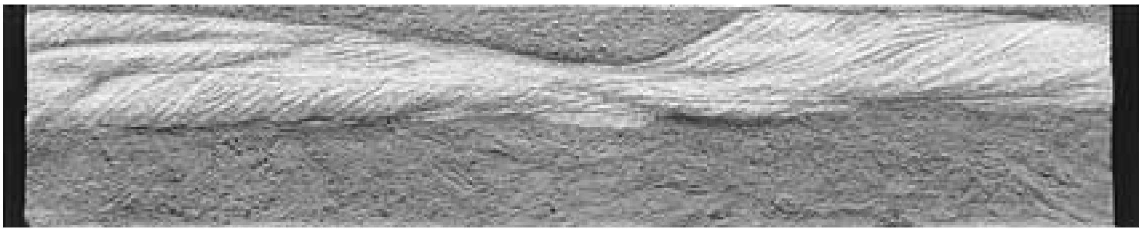

Fig. 6 Epoxy-resin sediment peel from deposit at the end of non-aggradational run BU8 (a replicate of run BU9; no photographs of peels from run BU9 are available). Flow is from right to left. Peel is 1 m long. Non-transported sediment (bottom) and filling (top) have been painted (dark grey) in order to enhance the cross-sets (light grey) formed by the migration of dunes with various height and trough-scour depth.

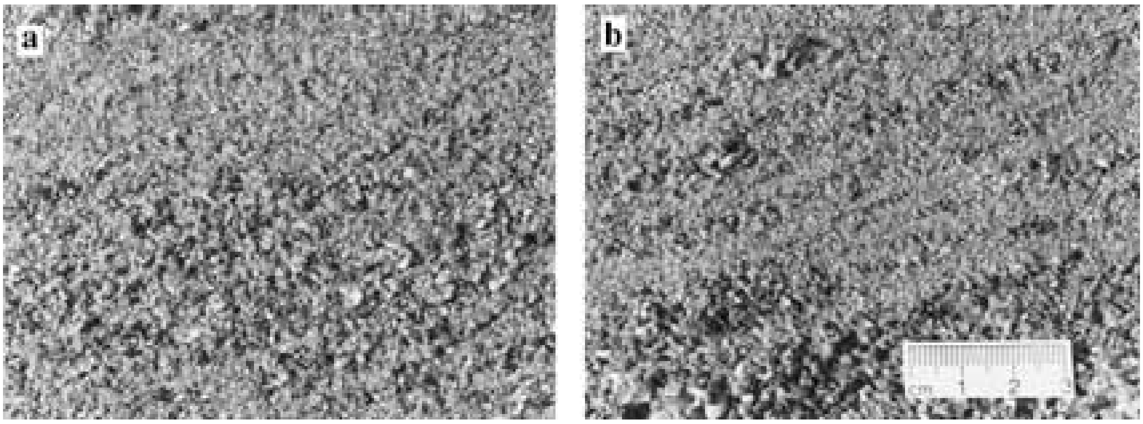

Fig. 7 Details of well-preserved cross-sets from (a) run SAFL 29 (low flow velocity) and (b) run SAFL 27 (high flow velocity). The upper boundaries of the cross-sets are seen as delicate lines at the level of the labels. Flow depth was approximately the same in both experiments. Flow direction is from right to left. In both cases, coarser grains gather at the base of the cross-set, but in the higher velocity run (b), coarser grains can be seen at top of cross-set and anywhere along the cross-strata, whereas at lower flow velocity (a), only finer grains compose the top part of the cross-set.

downstream, as does the thickness of individual cross-sets (Plate 2).

When dunes are well preserved, as they were in the experiment of Leclair (2000), vertical sorting can be observed within cross-sets. In matching flow-depth runs at low flow velocity (SAFL29, Fig. 7a) and high flow velocity (SAFL27, Fig. 7b), coarse grains can be observed at the base of cross-sets. At higher flow velocity (SAFL27), however, coarser grains are also seen along many cross-strata, and even at the very top of the cross-set (Fig. 7b). The presence of coarse grains at different depths seems to be due to the lateral variability in sediment transport in the case of more three-dimensional dunes at higher flow velocity (M. Kleinhans, personal communication, 2001). At lower flow velocity (SAFL29), coarse grains typically gather only at the bases of cross-sets (Fig. 7a). These findings may permit a qualitative interpretation of flow velocity from the internal textural features of the cross-sets. This information could be added to, or used instead of (depending on availability of data), similar interpretations from the ratio of mean cross-set thickness/mean cross-set length that decreases as the Froude number increases (Leclair, 2002).

From cross-sets to P_s

In the present analysis of bed-elevation probability distribution, the effect of flow depth on dune height (i.e. dune height tends to increase with flow depth, see Allen, 1982), and hence on the range of elevations in P_s curves, is removed by the use of dimensionless elevation, y/d. Therefore, a value for flow depth should be found in order to reconstruct the P_s curves from the characteristics of dune deposits. The estimation of flow depth from mean dune height is often based on commonly observed values, although it is well known that Yalin (1964) proposed a theoretical value of

$$h_m = 0.167d \tag{9}$$

Gill (1971) demonstrated that equation (9) can be derived from the Exner equation for sediment continuity, and is valid only for certain rates of sediment transport. The derivation proposed by Gill (1971, equation 16) is

$$h_m = (d/2n\alpha)(1 - (\tau_c/\tau_o)) \tag{10}$$

which includes controlling variables such as dune-shape factor α (our ts_m/h_m), sediment transport stage, $1 - (\tau_c/\tau_o)$, plus a constant and parameter n from a formula for bedload-transport rate. Following this approach, the relationship of dune height with flow depth, dune-shape factor and the present parameter for sediment-transport stage were investigated (Fig. 8), combined here in a variable G, defined as

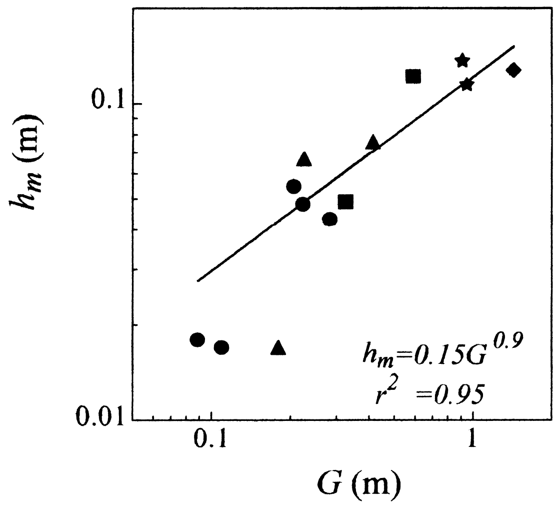

Fig. 8 Variation of mean dune height, h_m, with parameter $G = (d/\alpha)(1 - (\theta_c/\theta))$, with reduced major axis regression line. Similar symbols indicate experiments with similar flow depth, and show that mean dune height does not correlate well with flow depth, d, alone.

$$G = (d/\alpha)\Omega \tag{11}$$

In these experiments, the shape factor ranges from 0.41 to 0.74 and shows no clear relationship with the sediment-transport stage (Table 1). Roden *et al.* (1997) reported an increase in the shape factor, however, from 0.45 to 0.62, through a flood hydrograph in the Jamuna River, Bangladesh. It was not possible to include data from other studies in this functional analysis because no complete set of correspondent values of flow depth, dune height and shape factor, median grain size, shear stress and energy slope were found. The results from the reduced major axis regression on our combined experimental data set is

$$h_m = 0.15G^{0.9} \quad r^2 = 0.89 \tag{12}$$

or

$$G = 4.1h_m^{1.1} \quad r^2 = 0.91 \tag{13}$$

Equation (12) is close to Yalin's theory (equation 9) if $G \approx d$, i.e. if the value of the sediment-transport rate is about that of the dune-shape factor (see equation 11). Given that the dune-shape factor roughly ranges 0.5–0.7, and that $\theta_c \approx 0.045$ (Bridge & Bennett, 1992), the Shields parameter, θ, would vary from 0.09 to 0.15 in the case of $G \approx d$, and this is a limited range of θ for bedload-transport conditions. Such conditions may not lead to significant preservation of dune deposits (see above), hence high values of sediment transport stage (e.g. $\Omega \approx 0.9$) could be assumed if deposits are preserved at all. In this case, estimation of flow depth from cross-set thickness and dune height (using equation 13 then equation 11) could be constrained to the variability of the dune-shape factor ($\alpha \approx 0.5–0.7$). In addition, although the coefficient of determination, r^2, is high in the above equations, data with $G \leq 0.2$ m (runs A1, B1 and T10) depart markedly from the trend, indicating that there may be a cut-off value of G below which value dunes may not fully develop.

The lower and upper limits of the P_s curve also can be estimated from analysis of the distribution of the relative elevation of cross-set lower boundaries and cross-set thicknesses, respectively. Here, an example of how this could be achieved is presented (Fig. 9), using only one set of data on trough-scour depths of formative dunes from Leclair (2000). The distribution of the elevation

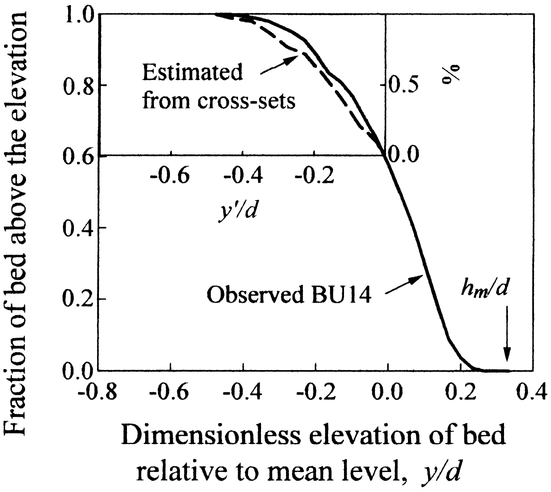

Fig. 9 Function P_s (y/d) for run BU14 shown along with function P_s (y'/d), where y' is the elevation of the lower boundary of cross-sets; y' is measured relative to the shallowest cross-set boundary (proxy for mean bed level) over the 3-m-long test section.

of dune trough-scour paths (e.g. as seen on Fig. 6) was measured relative to a surrogate of mean bed level, i.e. the elevation of the highest cross-set lower boundary (based on Fig. 1, it is very unlikely that the shallower cross-set boundary would be higher than mean bed level). Therefore, the probability distribution of these elevations (also made dimensionless) is a subsample of the P_s curve below $y/d = 0$. This subsample, y'/d, reproduces very well the observed lower limit, and the shape of the P_s curve on the low-elevation side (Fig. 9). The observed P_s curve has a higher fraction of bed above a given elevation because dune-trough scours are not the only bed-surface elevations lower than the mean. The upper limit can now be approximated from the estimation of mean dune height from cross-set thickness distribution (Leclair & Bridge, 2001). It seems that dunes with heights equal to, or larger than, the mean dune height ($h_i \geq h_m$) often cannot migrate higher than mean bed level, i.e. their trough could not be above mean bed level (i.e. for such dunes, $ts_i/h_i < 0$, see Fig. 5), owing to limitation by flow depth. The ratio of mean dune height over flow depth could then be an indicator of the upper limit of the P_s curve, and Fig. 3 shows clearly that, even for runs without any preserved deposit, this upper limit approximates the observed h_m/d (Table 1). At this time, insufficient results exist to identify clear controls for the rest of the P_s curve ($0 < y/d < h_m/d$).

DISCUSSION

Sediment transport stage and vertical sorting appear to be major controls on dune-bed topography, and hence on the shape of P_s. This is somehow expected, as the effects of the dimensionless bed shear stress on dune geometry, especially dune steepness, have been well known for decades (e.g. Yalin & Karahan, 1979). However, this large body of data is mostly about relationships between mean values (e.g. mean dune height and length), not about variability within a distribution, and hence, is less relevant for developing functional relationships for P_s, or for its interpretation. In the present experiments, it is shown that low variability of dune height and short-range P_s curves tend to be associated with low sediment transport stage (Table 1). Although it is clear that a higher shear stress means higher dunes, and thus a wider range of bed elevations exposed to the flow, the controls on dune height variability for comparable sediment transport stages remain an enigma: this is critical for predicting the left-hand side of the P_s curve with refinement. The results presented also extend the range of h_{sd}/ts_{sd} previously observed (i.e. 0.7–1.1; Leclair & Bridge, 2001) and used for predicting dune height from cross-set thickness. Table 1 shows that runs with $h_{sd}/ts_{sd} \approx 2.0$, which is a very commonly assumed value, actually produced no preserved cross-sets. This indicates that the reliability of the interpretative method proposed by Leclair & Bridge (2001), and hence the reconstruction of the P_s curve, should not be affected by these new results.

It is expected that in nature, the effect of sediment transport stage on the P_s curve would vary mostly with grain size (given y/d), as an increase in slope between different rivers would be counterbalanced by a decrease in grain size (Schumm, 1960; Williams, 1978). Moreover, for any given river, the changes in sediment mixture over time (for time scales relevant to dune migration) would be more important than changes in slope (Leopold & Maddock, 1953). However, the variability in vertical sorting is as important as mean grain size (which is used for computing sediment transport stage), as suggested by the differences between runs B2 and SAFL29 (Fig. 4, Table 1). The effect of energy slope, *s*, has not been elabotrated because values for all selected runs varied within only one order of magnitude, whereas D_{50} varied over two orders of magnitude (Table 1).

Non-dimensional PDFs directly facilitate comparison among experiments, but their use would also allow the prediction of several possible P_s curves for a range of h_m/d, e.g. depending on the range of dune shape factor and sediment transport stage used in equation (1), or on the estimated equilibrium status of dune height with flow conditions. High values of h_m/d (i.e. up to 0.37) are not only an effect of experimental conditions: they also have been observed in rivers with flow depths of 0.34–0.61 m (Gabel, 1993) and 7.8–9.1 m (Kostaschuk, 2000). In these two cases, the channels were rather straight (> 2 km long in Kostaschuk, 2000) and some of the lowest values (as small as 0.05) occurred in channel bends (Bridge & Jarvis, 1982). Otherwise, Mohrig & Smith (1996) observed $h_m/d \approx 0.25$ for dunes migrating over a bar. Apart from the obvious flow unsteadiness and non-uniformity in nature, it may well be that the 'fetch' in the alongstream direction, which is required for dunes to reach their equilibrium geometry, is often insufficient. It is likely that dunes from run SAFL33 would have continued to grow downstream if the flume had been longer. Therefore, when developing P_s curves, it may be necessary to adjust estimates of flow depth, depending on the estimated channel morphology and dune equilibrium with the flow in the problem of interest.

It is hoped that results from this research will help implement formulations for river bed variation in dune-forming conditions, such as the probabilistic Exner sediment continuity equation proposed by Parker *et al.* (2000). The present study provides a step towards developing a more robust theory that relates the dynamics of erosion and deposition during dune migration to bed topography, the geometry and vertical grain sorting of the deposits, and preserved cross-sets. For now, there is a particular need for more integrated data sets on grain-size selective transport rates, dune geometry, PDF of dune-trough elevation and of bed-surface elevation, vertical sorting profiles and the potential for preservation of the deposits.

CONCLUSION

Analysis of a combined set of data from two separate dune studies has identified certain controls on the bed-surface-elevation probability distribution (P_s), as this parameter relates to a new formulation of the Exner equation for sediment continuity. Moreover, this analysis also has revealed some indicators for reconstructing this distribution from preserved deposits.

Results show that the dimensionless bed shear stress, used here as a component of the sediment transport stage, and vertical sorting are the major controls on the range and shape of the nondimensional Ps curve. Low values of sediment transport stage produce P_s curves that are short-ranged and quite evenly spread around the average bed level. The presence of a coarse bed layer, which affects the composition of the transported sediment, has a similar effect on the P_s curve. Otherwise, P_s curves tend to be skewed toward the lowest values of bed-surface elevation. The entire distribution of grain size is important and seems to control the P_s curve near its lower limit, such that for similar values of sediment transport, the variability in dune height and trough-scour depth would depend on sorting (and initial vertical sorting). The P_s curves from dune-covered beds that are not in equilibrium with flow conditions do not reflect the actual value of sediment transport. Other controls on the overall shape of the P_s curve could not be clearly assessed.

The interpretation of the geometry of the preserved cross-sets can be used to partially reconstruct P_s curves. The low-elevation side of the curve can be approximated by analysis of the probability distribution of the elevation of the lower boundaries of cross-sets. The upper limit of P_s roughly corresponds to the ratio of mean dune height over flow depth. As a method for estimating mean dune height from the distribution of cross-set thicknesses already exists, here an empirical relationship for estimating flow depth from dune height and the dune shape factor, and sediment transport stage, is proposed. Moreover, grain sorting, among and within cross-sets, can allow a qualitative interpretation of flow velocity and sediment transport stage. It is hoped that this study will assist the development of a more comprehensive theory that ultimately can predict the characteristics of dunes and their deposits from data on sediment transport, or conversely, reconstruct sediment transport from the characteristics of preserved dune deposits.

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NOMENCLATURE

a	parameter = 1/β from gamma PDF of dune height distribution [L^{-1}]
d	mean flow depth [L]
D_{50}	median sediment grain size [L]
Fr	Froude number
g	gravitational acceleration [$L\ T^{-2}$]
G	compound variable controlling dune height [L]
h_i	height of an individual dune [L]
h_m	mean dune height [L]
h_{sd}	standard deviation of dune height [L]
S	mean water surface slope
s_m^{obs}, s_m^{pred}	(observed, predicted) mean cross-set thickness [L]
t	time [T]
ts_i	dune trough-scour depth below mean bed level for an individual dune [L] $ts_i < 0$ and > 0 when below and above mean bed level, respectively
ts_m	mean dune trough-scour depth below mean bed level [L]
ts_{sd}	standard deviation of dune trough-scour depth [L]
U	mean flow velocity [$L\ T^{-1}$]
y	bed-surface elevation relative to mean bed level [L]
y'	elevation of cross-set lower boundary relative to shallowest boundary [L]
λ	bed porosity
θ, θ_c	spatially averaged dimensionless bed shear stress, critical value at threshold of entrainment
ρ	fluid density [$M\ L^{-3}$]
σ	sediment density [$M\ L^{-3}$]
τ_o, τ_c	spatially averaged bed shear stress, critical value at threshold of entrainment [$M\ L^{-1}\ T^{-2}$]
Ω	sediment transport stage

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