Stability of the Synthesis Filter in Stereo Linear Prediction

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Abstract—**A scheme for stereo audio coding is proposed consisting of a 2-channel linear prediction stage in combination with a single rotator. It is argued that it combines different attractive features of existing stereo coding mechanisms. It is experimentally shown that for unequal orders of auto- and cross-predictors, the stability of the synthesis system cannot always be guaranteed. For equal orders, the Block-Levinson algorithm is applicable, which is the basis for the proof of the stability of the synthesis filter. It is shown that the forward and the backward error signal vectors appearing in a 2-channel lattice filter implementation are coupled via a 2-channel allpass filter. This latter finding can be used as the basis for an alternative proof of the stability of the synthesis system.**

Keywords—**Stereo Audio Coding; Linear Prediction; FIR Lattice; Principal Component Analysis; Stability.**

I. INTRODUCTION

In audio and speech coding, compression can be achieved by means of decorrelation methods such as *Linear Prediction* (LP), which removes *intra-channel correlations.* However, since audio signals usually consist of at least two channels which are often highly correlated, it is worthwhile to make use of *inter-channel correlations* as well. In contrast to known *stereo-LP* techniques [1-5], we propose a *symmetric* structure using LP-filters with a behavior that is in some way reminiscent of auditory filters [6], and then followed by a rotator performing *Principal Component Analysis* (PCA) [7] to create a main and side signal. The creation of the main and the side signal is similar to *Parametric Stereo Coding* [8, 9], while circumventing their inherent inability to reconstruct the original signal in the absence of quantization.

In this paper, we briefly review the existing stereo coding techniques and point out how our scheme in principle solves most of the problems present in the existing schemes. Experimentally we show that the synthesis system is not guaranteed to be stable when the orders of the auto- and the cross-predictors are different. We also give the proof for the stability for stereo-LP along with the filtering interpretation of the mathematics described in [10, 11].

The outline of this paper is as follows: Section II includes a review of the existing stereo coding techniques, and points out how our scheme solves most of the problems present in the existing schemes. Section III describes the details of our encoder along with the important steps of the calculation of prediction coefficients. In Section IV we present the experimental results which show that the synthesis system is not guaranteed to be stable when the orders of the auto- and the cross-predictors are different. Therefore, the case where the auto- and the cross-predictor orders are equal is considered in more detail. In particular, we present the *Normalized Block FIR Lattice* structure consistent with the *Normalized Block-Levinson* algorithm and prove that the forward and the backward error signal vectors are related by an allpass transfer. We conclude with a discussion in Section V.

II. STEREO CODING

Stereo coding aims at removing redundancy and irrelevancy from the stereo signal to attain lower bit-rates than the sum of the bit-rates of separate channels while maintaining the quality level. The following stereo coding tool already exists:

a. Mid-Side Stereo [12];

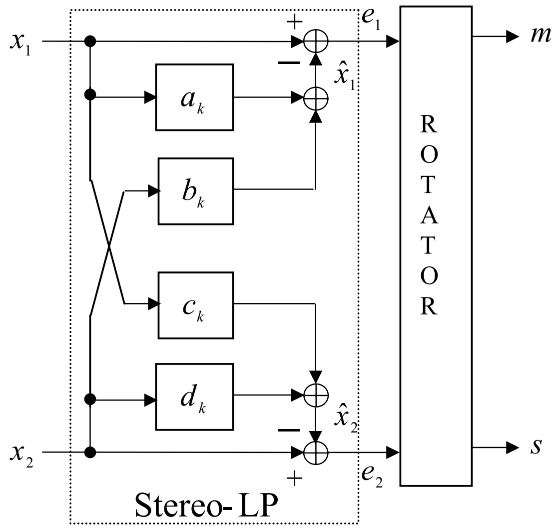

Figure 1 Redundancy removal by stereo-LP and a single rotator on a stereo signal.

b. Intensity Stereo [13];
c. Backward Adaptive Stereo-LP [1];
d. Linearly combining the left and the right channels into a complex signal and applying Warped LP to this complex signal [14];
e. Rotators over the total band or per frequency band [15];
f. Binaural Cue Coding [16, 17] and Parametric Stereo Coding [8, 9].

Out of the available techniques, the most promising technique for low bit-rate purposes appears to be Parametric Stereo Coding. The construction of a main and a side signal and very coarsely quantizing the side signal (or removing it altogether), yields the best balance in bit-rate/quality. In this respect it outperforms all the standard techniques. A problem that is inherent in this approach is that even in the absence of signal quantization, the original signal cannot be reconstructed perfectly due to the overlap-add procedure in the analysis and the synthesis systems.

Also, combining the left and the right channels in to a complex signal (as in [d]) is not equivalent to a general 2-channel structure and therefore is not able to attain maximum redundancy removal. Furthermore in [c], there is no identification or construction of main and side signal nor are the LP-filters based on psychoacoustic knowledge.

From the above description of the known techniques follows our problem statement. We desire a stereo coding technique subject to the following criteria:

- *Encoder and decoder form a system allowing perfect signal reconstruction in the absence of signal quantization and thus, near perfect reconstruction at high bit-rate end.*
- *The encoder constructs a main and a side signal similar to those provided by Binaural Cue Coding and Parametric Stereo Coding.*

An encoder which is able to meet these constrains is proposed in the next section.

III. STEREO-LP AND ROTATION

Our scheme uses a stereo-LP in combination with a single rotator. The transmitted data comprises the LP coefficients plus a rotation parameter. The encoder scheme is depicted in Figure 1. The left and the right signals, x_1 and x_2 are input to a stereo-LP stage yielding error signals e_1 and e_2. These signals are fed to a rotator producing the main and the side signals m and s, respectively. The rotator performs a matrix multiplication described by

$$\begin{bmatrix} m \\ s \end{bmatrix} = \begin{bmatrix} \cos(\alpha_0) & \sin(\alpha_0) \\ -\sin(\alpha_0) & \cos(\alpha_0) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \qquad (1)$$

where α_0 is optimal rotation angle. The decoder implements the reverse system. First, the rotation is undone by an inverse rotation, and followed by a stereo-LP synthesis filter.

The analysis system ensures that the spectra of the signals are flattened and thus the crosscorrelation function associated with e_1 and e_2 is minimized except for lag zero. This is a situation which a rotator is able to cope with, and therefore can be used to construct the main and the side signal, resulting in two spectrally flat, uncorrelated signals m and s.

Let us consider the stereo-LP scheme as shown in Figure 1. We have the two channels of the stereo signal x_1 (left channel) and x_2 (right channel), and a set of regressor signals given by $y_{(11)k}, k = 1,2,\ldots,K_a$, and $y_{(12)k}, k = 1,2,\ldots,K_b$ for the channel x_1. Similarly, for the channel x_2 we have the regressor signals $y_{(21)k}, k = 1,2,\ldots,K_c$, and $y_{(22)k}, k = 1,2,\ldots,K_d$. The regressor signals are related to inputs x_1 and x_2 by linear filtering, thus

$$
\begin{aligned}
Y_{(11)k}(z) &= G_k(z)X_1(z); k = 1,2,\ldots,K_a \\
Y_{(12)k}(z) &= G_k(z)X_2(z); k = 1,2,\ldots,K_b \\
Y_{(21)k}(z) &= G_k(z)X_1(z); k = 1,2,\ldots,K_c \\
Y_{(22)k}(z) &= G_k(z)X_2(z); k = 1,2,\ldots,K_d
\end{aligned} \tag{2}
$$

where $X_1(z)$, $X_2(z)$, $Y_{(11)k}(z)$, $Y_{(12)k}(z)$, $Y_{(21)k}(z)$, and $Y_{(22)k}(z)$ are the z-transforms for the signals x_1, x_2, $y_{(11)k}$, $y_{(12)k}$, $y_{(21)k}$, and $y_{(22)k}$ respectively, and $G_k(z)$ is the k^{th} stable transfer function. For conventional stereo-LP we have

$$G_k(z) = z^{-k}. \tag{3}$$

For perceptual reasons the *Laguerre*-based *Pure Linear Prediction* (L-PLP) scheme can be used [6]. This changes the filtering operation $G_k(z)$ to

$$
\begin{aligned}
G_k(z) &= z^{-1}H_k(z) \\
&= z^{-1}\frac{\sqrt{1-\lambda^2}}{1-z^{-1}\lambda}\left(\frac{z^{-1}-\lambda}{1-z^{-1}\lambda}\right)^{k-1},
\end{aligned} \tag{4}
$$

where $H_k(z)$ are stable and causal Laguerre filters with $\lambda \in \Re$ and $|\lambda| < 1$. For this case, the spectral flattening is effectively done on a psychoacoustic relevant frequency scale for an appropriate choice of λ [18]. Note that conventional LP is a special case of L-PLP where $\lambda = 0$. In the rest of this article, the discussion addresses the situation $G_k(z) = z^{-k}$. However, the results can be easily extended to the case of the L-PLP scheme in a similar way as was done for the single channel case [6].

A. Prediction of the left channel

The prediction $\hat{x}_1$ of x_1 is derived from the signals $y_{(11)k}$ and $y_{(12)k}$ by

$$
\begin{aligned}
\hat{x}_1 &= \sum_{k=1}^{K_a} a_k y_{(11)k} + \sum_{k=1}^{K_b} b_k y_{(12)k} \\
&= \sum_{k=1}^{K_a} a_k x_1(n-k) + \sum_{k=1}^{K_b} b_k x_2(n-k),
\end{aligned} \tag{5}
$$

where a_k and b_k are the auto- and the cross-predictor coefficients, respectively. The estimation of the prediction coefficients can be done by minimizing a suitable norm of the residual signal $\|e_1\|$, where $e_1 = x_1 - \hat{x}_1$. Usually they are optimized to minimize the mean squared prediction error e_1. Accordingly, we take the deterministic measure J as criterion

$$J = \sum_{n=-\infty}^{\infty} |e_1(n)|^2. \tag{6}$$

Optimization with respect to a_k and b_k results in the following set of equations

$$r_{11}(k) = \sum_{l=1}^{K_a} a_l r_{11}(l-k) + \sum_{l=1}^{K_b} b_l r_{12}(l-k); \forall k \in \{1,\ldots,K_a\}, \tag{7}$$

$$r_{12}(k) = \sum_{l=1}^{K_a} a_l r_{21}(l-k) + \sum_{l=1}^{K_b} b_l r_{22}(l-k); \forall k \in \{1,\ldots,K_b\}, \tag{8}$$

involving the correlation function between the channels defined by $r_{pq}(k-l) = \sum_n x_p(n-l)x_q(n-k)$, where $p,q = 1,2$. The merging of Equations (7) and (8) yields in matrix form the following set of equations

$$\begin{bmatrix}\mathbf{r}_{11}\\ \mathbf{r}_{12}\end{bmatrix} = \begin{bmatrix}\mathbf{R}_{11} & \mathbf{R}_{12}\\ \mathbf{R}_{21} & \mathbf{R}_{22}\end{bmatrix}.\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = \mathbf{R}_1\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix}, \tag{9}$$

with the correlation and the coefficient vectors as

$$
\begin{aligned}
\mathbf{r}_{11} &= \left[r_{11}[1] \quad r_{11}[2] \quad \ldots \quad r_{11}[K_a]\right]^T, \\
\mathbf{r}_{12} &= \left[r_{12}[1] \quad r_{12}[2] \quad \ldots \quad r_{12}[K_b]\right]^T, \\
\mathbf{a} &= \left[a_1 \quad a_2 \quad \ldots \quad a_{K_a}\right]^T, \\
\mathbf{b} &= \left[b_1 \quad b_2 \quad \ldots \quad b_{K_b}\right]^T,
\end{aligned} \tag{10}
$$

the autocorrelation matrices $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$ as

$$
\begin{aligned}
\mathbf{R}_{11} &= \begin{bmatrix} r_{11}(0) & \cdots & r_{11}(K_a-1) \\ \vdots & \cdots & \vdots \\ r_{11}(1-K_a) & \cdots & r_{11}(0)\end{bmatrix}, \\
\mathbf{R}_{22} &= \begin{bmatrix} r_{22}(0) & \cdots & r_{22}(K_b-1) \\ \vdots & \cdots & \vdots \\ r_{22}(1-K_b) & \cdots & r_{22}(0)\end{bmatrix},
\end{aligned} \tag{11}
$$

and the crosscorrelation matrices $\mathbf{R}_{12}$ and $\mathbf{R}_{21}$ as

$$
\begin{aligned}
\mathbf{R}_{12} &= \begin{bmatrix} r_{12}(0) & \cdots & r_{12}(K_b-1) \\ \vdots & \cdots & \vdots \\ r_{12}(1-K_a) & \cdots & r_{12}(K_b-K_a)\end{bmatrix}, \\
\mathbf{R}_{21} &= \begin{bmatrix} r_{21}(0) & \cdots & r_{21}(K_a-1) \\ \vdots & \cdots & \vdots \\ r_{21}(1-K_b) & \cdots & r_{21}(K_a-K_b)\end{bmatrix}.
\end{aligned} \tag{12}
$$

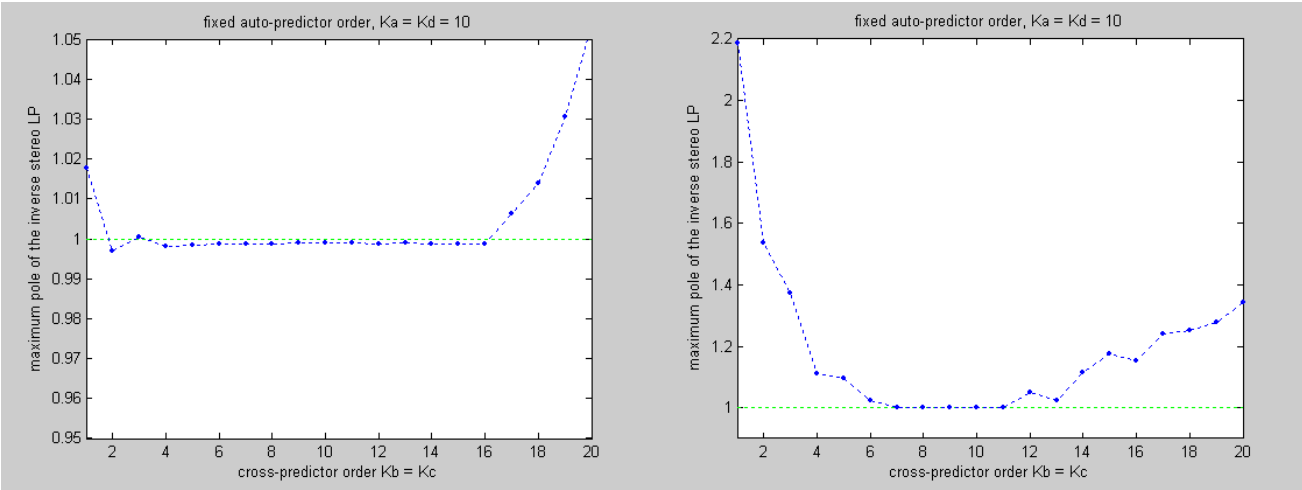

Figure 2 The maximum pole of the synthesis filter as a function of the cross-predictor orders for Trumpet (left) and Suzanne Vega (right); for fixed auto-predictor orders.

The autocorrelation matrices $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$ are *symmetric* Toeplitz matrices, whereas the crosscorrelation matrices $\mathbf{R}_{12}$ and $\mathbf{R}_{21}$ are *mutually symmetric* matrices since $r_{pq}(k-l) = r_{qp}(l-k)$ and therefore

$$\mathbf{R}_{21} = \mathbf{R}_{12}^{T}. \tag{13}$$

The optimal auto- and the cross-predictor coefficients $\mathbf{a}$ and $\mathbf{b}$ are defined by Equation (9).

B. Prediction of the right channel

In contrast to known stereo redundancy reduction techniques [1-5], we propose a symmetric structure using LP-filters, i.e. for the right channel we can develop the same kind of Equations as in Equations (5) to (13) to calculate the auto- and the cross-predictor coefficients d_k and c_k. In the special case where $K_a = K_b = K_c = K_d$, the sets of equations for the optimal coefficients a_k, b_k, c_k, and d_k can be merged into set of equations involving a Block-Toeplitz structure such that it is also possible to use the *Block-Levinson* algorithm [10, 11, 19].

C. Rotation

In the stereo-LP analysis system, we minimize the powers of e_1 and e_2 and the absolute value of the cross-powers. This results in spectrally flat error signals e_1 and e_2 and a spectrally flat cross-power. Subsequently we rotate the signals e_1 and e_2 such that these signals are orthogonal. Calculation of the optimal rotation angle can be done in the usual way i.e., by PCA [7]. The optimal angle is typically defined as an angle which produces a maximum of a weighted squared sum of the signal m, which automatically induces a minimum for the weighted squared sum of s.

It is important to note that the optimal value of α_0 is ambiguous. In order to select a unique solution, we consider continuity of the optimal solution over frames i.e. we select from the possible optimal values for the current frame the one which is the closest to the optimal value of the previous frame.

IV. STABILITY ANALYSIS

A. Experimental Observation

In this section, we show that the stability of the synthesis filter of the stereo-LP is not guaranteed for unequal auto- and cross-predictor orders. We consider two audio files in our experiments. One is a mono like (Suzanne Vega) and the other is a stereo like (Trumpet). All these files are of CD format (i.e. 44.1 kHz, 16 bits/sample). Hanning-windowed segments of 2048 samples were used for calculation of the optimal prediction coefficients. The order of auto-predictors K_a and K_d is set to 10. Next, we varied the order of the cross-predictors K_b and K_c of the stereo-LP from 1 to

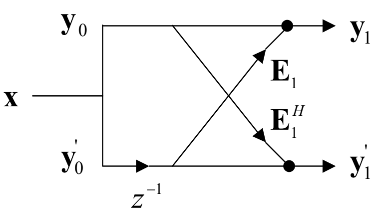

Figure 3 First stage of a Block FIR Lattice filter.

20 and determined the maximum pole of the synthesis system.

Figure 2 presents the maximum magnitude of the pole of the synthesis system obtained using different settings of the cross-predictor order. The points underneath the horizontal dotted line indicate stability of the synthesis system.

We note that for the Trumpet, the synthesis filter is stable for a cross-predictor order of 2, and 4-16 for a fixed auto-predictor order of 10. And, for the Suzanne Vega, it is stable for the cross-predictor orders 7-11 for a fixed auto-predictor order of 10. The experiments using various other tracks show that the synthesis system is always stable when the order of the cross-predictors equals that of the auto-predictors. It is clear that for the case $(K_a = K_d) \neq (K_b = K_c)$, the stability is not always guaranteed.

B. Theoretical Proof

This experimental observation motivated the authors to identify the basic equations of the Block-Levinson algorithm computing the prediction filters with the recurrence relations for the orthogonal polynomials matrices appearing in this algorithm [10, 11]. Positive definite Block-Toeplitz matrices occur as autocorrelation matrices of the multichannel process. The stability of such an inverse has already been proved in [19]. Here we consider a different approach to gain deeper insights. It is shown that the algorithm has an interesting interpretation as a Block FIR Lattice filter type realization for matrix-valued reflection coefficients.

Most of the results of this paper are natural extensions of those obtained for the scalar case, although the point of view is somewhat different. The whole subject originates from the Block-Levinson algorithm to compute the prediction polynomials and the associated reflection coefficients matrices for the specific case $K_a = K_b = K_c = K_d = K.$

For equal orders of auto- and the cross-predictors we obtain a sequence of complex 2×2 matrices $\mathbf{C}_k$, with $k = 0, \pm 1, \pm 2, \ldots, \pm K,$ having the property that all Block-Toeplitz matrices

$$\mathbf{\Gamma}_k = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_{-1} & \cdots & \mathbf{C}_{-k} \\ \mathbf{C}_1 & \mathbf{C}_0 & \cdots & \mathbf{C}_{1-k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_k & \mathbf{C}_{k-1} & \cdots & \mathbf{C}_0 \end{bmatrix}, \qquad (14)$$

are Hermitian, thus satisfying $\mathbf{C}_{-k} = \mathbf{C}_k^H$. More specifically $\mathbf{C}_k$ is given by

$$\mathbf{C}_k = \begin{bmatrix} r_{11}(k) & r_{12}(k) \\ r_{21}(k) & r_{22}(k) \end{bmatrix}, \qquad (15)$$

where the elements of the matrix $\mathbf{C}_k$ are the same as the ones described in Section III. It should be mentioned that the Block-Toeplitz matrix $\mathbf{\Gamma}_k$ appearing in such a framework is always positive definite but may be singular. Here we assume it to be positive definite, but not singular.

The Block-Levinson algorithm can compute the forward and the backward auto- and the cross-predictor coefficients stored in the 2×2 reflection coefficient matrices $\mathbf{E}_k$ and $\mathbf{E}_k^{'}$. The prime denotes the *backward* reflection coefficient matrix. In contrast to the single channel situation, we note that $\mathbf{E}_k$ and $\mathbf{E}_k^{'}$ are not directly related; i.e. information of $\mathbf{E}_k$ itself is not sufficient to construct $\mathbf{E}_k^{'}$. Similar to the classical Levinson-Durbin recursion, the Block-Levinson algorithm leads to an FIR Lattice filter, but now a Block FIR Lattice filter. The first stage of this is shown in Figure 3, where the 2×1 vectors $\mathbf{y}_1$ and $\mathbf{y}_1^{'}$ denote the forward and the backward prediction error, respectively, after the first stage. The elements of 2×1 input vector $\mathbf{x}$ consist of signals x_1 and x_2 of Figure 1.

Based on Figure 3, we formulate the following problem. We desire to prove that the transfer $\mathbf{y}_1 \to \mathbf{x}$ is invertible, i.e. the inverse system of the first-order stereo-LP is stable. But before we proceed on with the proof, let use review a few useful identities from the Block-Levinson algorithm. Firstly,

$$\mathbf{R}_{k-1}\mathbf{E}_k^{'} = \mathbf{E}_k^H \mathbf{R}_{k-1}^{'}, \qquad (16)$$

where $\mathbf{R}_k$ and $\mathbf{R}_k^{'}$ are positive definite 2×2 forward and backward innovation variance matrices, whose recurrence relations are given by

$$\mathbf{R}_k = \mathbf{R}_{k-1}(\mathbf{I}-\mathbf{E}'_k\mathbf{E}_k) = \mathbf{R}_{k-1} - \mathbf{E}_k^H\mathbf{R}'_{k-1}\mathbf{E}_k,$$
$$\mathbf{R}'_k = \mathbf{R}'_{k-1}(\mathbf{I}-\mathbf{E}_k\mathbf{E}'_k) = \mathbf{R}'_{k-1} - \mathbf{E}_k^{'H}\mathbf{R}_{k-1}\mathbf{E}'_k, \quad (17)$$

where $\mathbf{I}$ denotes a 2×2 identity matrix. It is important that the reader should not confuse $\mathbf{R}_k$ with the correlation matrices used in Section III. Also, it is important to note that $\mathbf{R}_0 = \mathbf{C}_0$.

It follows from Equation (17) that the eigenvalues λ_1 and λ_2 of the matrices $\mathbf{E}'_k\mathbf{E}_k$ and $\mathbf{E}_k\mathbf{E}'_k$ are real and less than unity, and also that the eigenvalues of both matrices are the same. The property $\lambda_i(\mathbf{E}_k\mathbf{E}'_k) < 1$ for $1 \le k \le K$, together with $\lambda(\mathbf{R}_0) > 0$, can be used as a criterion for the positive definiteness of a given Hermitian Block-Toeplitz matrix $\mathbf{\Gamma}_k$.

Normalized Reflection Matrices

The forward and the backward reflection matrices can be rewritten to the one using *normalized reflection coefficient matrices.* Therefore, we consider the factorizations of $\mathbf{R}_k$ and $\mathbf{R}'_k$ in the form [10, 11]

$$\mathbf{R}_k = \mathbf{M}_k^H\mathbf{M}_k,\ \mathbf{R}'_k = \mathbf{M}_k^{'H}\mathbf{M}'_k, \quad (18)$$

for suitable 2×2 normalizing matrices $\mathbf{M}_k$ and $\mathbf{M}'_k$. Generally, $\mathbf{M}_k$ and $\mathbf{M}'_k$ are calculated within unitary left factors. Next we define the normalized reflection coefficient matrix ξ_k by

$$\xi_k = \mathbf{M}'_{k-1}\mathbf{E}_k\mathbf{M}_{k-1}^{-1} = \left(\mathbf{M}_{k-1}^{'H}\right)^{-1}\mathbf{E}_k^{'H}\mathbf{M}_{k-1}^H. \quad (19)$$

The forward and the backward normalized reflection coefficient matrices are now directly coupled by $\xi'_k = \xi_k^H$. Using Equation (17) in combination with Equations (18) and (19) gives

$$\left(\mathbf{M}_{k-1}^H\right)^{-1}\mathbf{M}_k^H\mathbf{M}_k\left(\mathbf{M}_{k-1}\right)^{-1} = \mathbf{I} - \xi_k^H\xi_k. \quad (20)$$

From Equation (20) one can prove that the spectral norm of ξ_k is less than unity or, equivalently, that $\mathbf{I}-\xi_k^H\xi_k$ is positive definite.

Define $\mathbf{M}_k$ from $\mathbf{M}_{k-1}$ in such a way that $\mathbf{M}_k\mathbf{M}_{k-1}^{-1}$ is a positive definite Hermitian matrix satisfying Equation (20). Thus we have

$$\mathbf{M}_k = \left(\mathbf{I}-\xi_k^H\xi_k\right)^{1/2}\mathbf{M}_{k-1}. \quad (21)$$

$$\mathbf{M}'_k = \left(\mathbf{I}-\xi_k\xi_k^H\right)^{1/2}\mathbf{M}'_{k-1}. \quad (22)$$

The initial values are taken to be $\mathbf{M}_0 = \mathbf{M}'_0 = \mathbf{C}_0^{1/2}$.

Using Equations (18) to (22), the network of Figure 3 is reorganized into a form as shown in Figure 4. We call this reorganized network as the *Normalized Block FIR Lattice filter*.

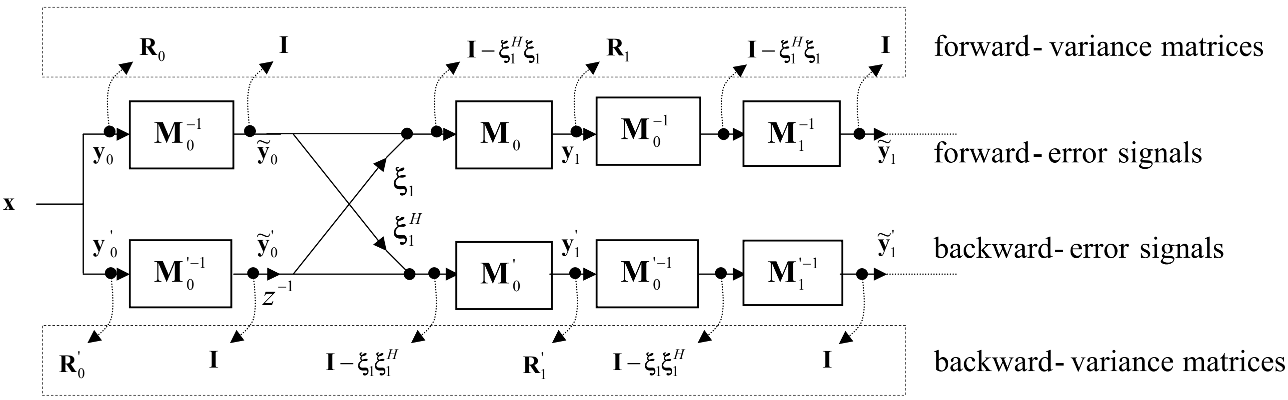

Figure 4 First stage of the Normalized Block FIR Lattice filter.

Obviously, the cascade of the blocks $\mathbf{M}_0, \mathbf{M}_0^{-1}$ and $\mathbf{M}'_0, \mathbf{M}_0^{'-1}$ can be omitted. These are inserted in order to show the relationship with the unnormalized version of Figure 3.

Now, the transfer $\tilde{\mathbf{y}}_1 \to \tilde{\mathbf{y}}'_1$ can be proved to be an allpass. The transfer $\tilde{\mathbf{y}}_1 \to \tilde{\mathbf{y}}'_1$ is given by

$$\mathbf{A}_1(z) = (\mathbf{M}'_1)^{-1}\left(z^{-1}\mathbf{I}+\xi_1^H\right)\left(\mathbf{I}+z^{-1}\xi_1\right)^{-1}\mathbf{M}_1. \quad (23)$$

The transfer $\mathbf{A}_1(z)$ is an allpass if and only if $\mathbf{A}_1^H(e^{j\omega})\mathbf{A}_1(e^{j\omega}) = \mathbf{I}$. Using singular value decomposition we can decompose ξ_1 as

$$\xi_1 = \mathbf{USV}^H, \quad (24)$$

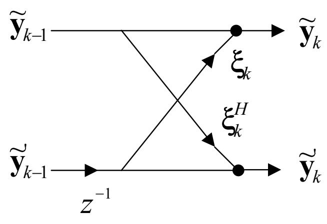

Figure 5 The k^{th} stage of a Normalized Block FIR Lattice filter.

where $\mathbf{U}$ and $\mathbf{V}$ are 2×2 unitary matrices, and $\mathbf{S}$ is a 2×2 singular value matrix of ξ_1. Furthermore, we note from the fact that the spectral norm of ξ_k is less than unity, we have

$$\left[\mathbf{I}-\mathbf{S}\mathbf{S}^H\right]^{-1}=\mathbf{I}+\mathbf{S}\mathbf{S}^H+(\mathbf{S}\mathbf{S}^H)^2+\ldots. \qquad (25)$$

Substituting Equation (24) into Equation (23), and using Equation (25), we get the desired identity $\mathbf{A}_1^H(e^{j\omega})\mathbf{A}_1(e^{j\omega})=\mathbf{I}.$

Continuing for the second-order section we see that

$$\mathbf{A}_2(z)=(\mathbf{M}_2^{'})^{-1}\left(z^{-1}\mathbf{A}_1(z)+\xi_2^H\right)\left(\mathbf{I}+z^{-1}\xi_2\mathbf{A}_1(z)\right)^{-1}\mathbf{M}_2. \qquad (26)$$

This is an allpass as well, since an allpass transfer within an allpass transfer is an allpass function. Continuing in this fashion, the k^{th} transfer function is also an allpass transfer. Hence, $\mathbf{y}_k\rightarrow\mathbf{y}_k^{'}$ is an allpass transfer.

The transfer $\mathbf{A}_k(z)$ is a stable allpass filter if $\mathbf{I}+z^{-1}\xi_k\mathbf{A}_{k-1}(z)$ is regular for $|z|>1$. Since ξ_k is a contracting matrix, and assuming $z^{-1}\mathbf{A}_{k-1}(z)$ is a stable allpass filter implies that $\mathbf{A}_k(z)$ is also a stable allpass filter with the magnitudes of its eigenvalues less than 1 for $|z|>1$. From this inductive reasoning together with $\mathbf{A}_0=\mathbf{I},$ it follows that $\mathbf{A}_k(z)$ is a stable allpass filter.

Now let us consider $\tilde{\mathbf{y}}_{k+1}$. In the $z-$domain we have

$$\begin{aligned}\tilde{\mathbf{Y}}_{k+1}(z)&=\tilde{\mathbf{Y}}_k(z)+z^{-1}\xi_{k+1}\tilde{\mathbf{Y}}_k^{'}(z)\\&=(\mathbf{I}+z^{-1}\xi_{k+1}\mathbf{A}_k(z))\tilde{\mathbf{Y}}_k(z),\end{aligned} \qquad (27)$$

where $\tilde{\mathbf{Y}}_k(z)$ is the input to the $(k+1)^{th}$ stage. In view of fact that ξ_{k+1} is a contracting matrix and that $\mathbf{A}_k(z)$ is a stable allpass, the transfer $\mathbf{I}+z^{-1}\xi_{k+1}\mathbf{A}_k(z)$ is a regular matrix for $|z|>1$. Consequently, the filtering $\mathbf{y}_{k+1}\rightarrow\mathbf{y}_k$ is stable. Since this holds for every section $k,$ the total synthesis system $\mathbf{y}_K\rightarrow\mathbf{x}$ is stable.

V. CONCLUSIONS

A 2-channel stereo-LP scheme cascaded by a rotator was proposed. It was argued that this might be a proper concept for stereo coding of audio signals since it combines several attractive features of known coding mechanisms. It was shown that the stability of the synthesis system is not guaranteed for unequal auto- and cross-predictor orders. For equal orders, a Block FIR Lattice filter was introduced to interpret the variables that appear in the Block-Levinson algorithm, in particular for the version working with normalized reflection coefficient matrices. It was shown that the forward and the backward prediction error vectors occurring in the Block FIR Lattice filter are coupled via a 2-channel allpass filter. It was shown that this enables an alternative proof of the stability of the synthesis filter.

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